

# A simple experiment for determining Verdet constants using alternating current magnetic fields

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A simple experiment suitable for a senior undergraduate and graduate laboratory on the measurement of Faraday rotation using ac magnetic fields is described. The apparatus was used to measure the wavelength dependence of Verdet constants for several materials. A concentration-dependent study on ferric chloride water solution was also carried out to demonstrate that the diamagnetic and paramagnetic contributions to Faraday rotation have opposite signs.

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## I. INTRODUCTION

Faraday rotation<sup>1</sup> is the rotation of the plane of polarization of light due to magnetic-field-induced circular birefringence in a material. In a nonabsorbing or weakly absorbing medium a linearly polarized monochromatic light beam passing through the material along the direction of the applied magnetic field experiences circular birefringence, which results in rotation of the plane of polarization of the incident light beam. The angle of rotation  $\theta$  can be expressed as  $\theta = VBL = \pi\Delta nL/\lambda$ , where  $\Delta n$  is the magnitude of circular birefringence (a difference in refractive index of left and right circularly polarized light in the medium),  $L$  is the length of the medium traversed,  $\lambda$  is the wavelength of light in vacuum, and  $B$  is the magnetic field. The constant  $V$  is a material property and gives a quantitative measure of the Faraday rotation ability of the material and is called the Verdet constant. Faraday rotation is commonly used in fabrication of optical isolators<sup>2</sup> to prevent unwanted backreflections. Faraday rotation measurements can also be used to infer the susceptibility of materials and measure carrier densities<sup>3</sup> in semiconductors if the effective mass of the carrier is known.

Many investigations have been carried out to measure the Verdet constants of materials using dc magnetic fields.<sup>4-7</sup> Since Faraday rotation combines many fundamental elements of polarization optics with electromagnetism, dc magnetic-field-based Faraday rotation experiments have also been introduced to the undergraduate physics laboratory to examine the Faraday rotation and determine the Verdet constants of samples.<sup>5,6</sup> However, the dc magnetic-field-based measurement requires an expensive and bulky electromagnet and a costly high current power supply to produce magnetic fields of the order of a kilogauss or higher due to the small values of the Verdet constants for paramagnetic and diamagnetic materials (of the order  $10^{-3}$  to  $10^{-2}$  min/G cm). Measurement of the Verdet constants with an ac magnetic field has been recently reported.<sup>7</sup> In this paper, we describe a simple experimental setup suitable for a senior or graduate laboratory to investigate Faraday rotation and determine the Verdet constant using an ac magnetic field. The experiment uses a lock-in amplifier to measure the Faraday rotation angle and introduces students to the utility of the phase sensitive detection technique to extract a signal normally buried

in noise. The Verdet constants of several materials commonly available in the laboratory were determined at different wavelengths and compared with published data.<sup>8,9</sup>

## II. EXPERIMENT

The experimental setup is schematically shown in Fig. 1. The conventional large electromagnet is replaced with two 500-turn coils (wound with enamelled copper wire 1 mm in diameter). These coils were routinely used in our undergraduate labs for demonstrating concepts relating to inductances and resonant circuits. Each coil has an air core with a cross section of 4 cm $\times$ 4 cm and a length of 7 cm. Each of these coils has an inductance of 0.005 H and a resistance of 2.5  $\Omega$  allowing a maximum current of 2.5 A. The spacing between the coils is 3 cm. The power source to drive each of the coils generating the ac field was a car stereo amplifier capable of supplying 50-W per channel and was bought from a discount store for \$30.00. A 13.8-V 10-A power supply was used as the power source for the amplifier. Two 1- $\mu$ F capacitors (rated for 300 V) were connected in series with each of the coils to resonate the circuits at a convenient drive frequency which was chosen to be in the vicinity of 2 kHz. The resonant circuit helps cancel current limiting due to the inductive impedance of the coils and makes it possible to generate larger ac magnetic fields. It is important to bear in mind that due to the resonant excitation the voltage across

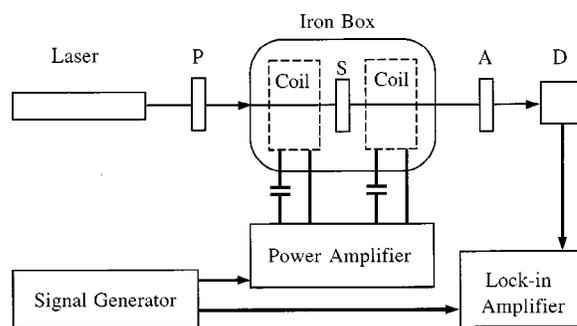


Fig. 1. Schematic diagram of the setup for measuring the Faraday rotation using ac magnetic field. P: polarizer; S: sample; A: analyzer; D: photodetector.

the capacitor and inductors can get as high as 200 V and care should be taken to make sure that these components are appropriately insulated or contained in a closed box to avoid a possible shock during operation. The sinusoidal output of a signal generator was fed into the two input channels of the audio amplifier and the frequency was scanned until resonance was obtained at 2.2 kHz. Typically, the ac magnetic field was adjusted to be in the range of 20–40 G rms. The magnitude of the magnetic field can be controlled by adjusting the magnitude of the input signal or the frequency of the signal generator so as to move slightly away from the resonance peak. The magnetic field was measured using a Hall probe ac gaussmeter. If a gaussmeter is not available, the magnetic field can still be determined by measuring the Faraday rotation of a material whose Verdet constant is accurately known. We shall further elaborate on this in Sec. IV.

In this experiment, a polarizer (inexpensive polarizers with extinction ratios of 1:1000 work fine) is used to obtain a linearly polarized light beam that passes through the sample placed between the two coils. The plane of polarization of the light beam passing through the sample under the influence of the ac magnetic field oscillates periodically. This oscillation of the plane of polarization is converted by another polarizer (as analyzer) that is set at 45° with respect to the input polarizer to an oscillation of light intensity which is detected by a photodetector and a lock-in amplifier (Stanford Research Systems SR510 single-phase analog lock-in amplifier with a frequency range of 0.5 Hz–100 kHz). The photodetector used here is a silicon-based low-power detector (Newport 818-SL) which has a broad spectral response of 400–1100 nm and a rise time about 2  $\mu$ s. The Faraday rotation angle is experimentally measured and the Verdet constant can be easily determined if the value of the magnetic field and sample thickness are known. Section III discusses some of the theoretical aspects underlying the experiment.

### III. THEORY

The linearly polarized light incident on the sample is assumed to be propagating in the  $z$  direction and polarized along the  $x$  axis. The electric field of the light beam can be expressed in Jones matrix form as

$$E_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} A_0 \exp(-i\omega t + ikz), \quad (1)$$

where  $A_0$  is the amplitude of the electric field of the beam. After passing through the sample, the polarization of the light is rotated by a small angle  $\theta$  and can be represented by

$$E = \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} A_0 \exp(-i\omega t + ikz). \quad (2)$$

The light then traverses through an analyzer which is set at an angle of  $\phi$  with respect to the polarizer. The electric field of the light beam is now described as

$$E = \begin{pmatrix} \cos(\phi - \theta) \cos \phi \\ \cos(\phi - \theta) \sin \phi \end{pmatrix} A_0 \exp(-i\omega t + ikz) \quad (3)$$

and the light intensity measured by the detector is given by

$$I = \cos^2(\phi - \theta) A_0^2. \quad (4)$$

In order to achieve a maximum modulation of the light intensity ( $\Delta I$ ), the analyzer needs to be set at an optimal angle. By taking the first derivative of  $I$  with respect to  $\theta$ , one gets

$$\frac{\partial I}{\partial \theta} = \sin 2(\phi - \theta) A_0^2. \quad (5)$$

Since the angle  $\theta$  is very small ( $\theta \ll 1^\circ$ ), one can clearly see that when  $\phi = 45^\circ$  maximum  $\Delta I$  can be obtained. With the analyzer set at 45°, the intensity measured by the detector as expressed by Eq. (4) can be simplified:

$$I = \frac{1}{2}(1 + 2\theta) A_0^2. \quad (6)$$

Since the ac magnetic field is sinusoidal  $B = B_0 \sin(\Omega t)$  and  $\theta$  is proportional to  $B$ , the rotation angle can be written in the form of  $\theta = \theta_0 \sin(\Omega t)$ . Then Eq. (6) can be rewritten as

$$I = \frac{1}{2}(1 + 2\theta_0 \sin(\Omega t)) A_0^2 = I_0 + \Delta I \sin(\Omega t). \quad (7)$$

By measuring the relative change of the light intensity  $\Delta I/I_0$ , the amplitude of the angle of rotation can be easily determined to be

$$\theta_0 = \frac{1}{2} \frac{2\theta_0 A_0^2}{A_0^2} = \frac{1}{2} \frac{\Delta I}{I_0}. \quad (8)$$

The Verdet constant of the material is then determined using the relation  $\theta_0 = VB_0L$ , where  $\theta_0$ ,  $L$ , and  $B_0$  are experimentally determined. It is important to point out that it is not difficult to measure a ratio  $\Delta I/I_0$  of the order  $10^{-5}$  using a phase sensitive technique. A rotation of the plane of polarization of the order of  $10^{-5}$  rad is thus measurable by this technique due to the elimination of broadband noise by the lock-in amplifier. A rotation of  $10^{-5}$  rad corresponds to a difference of the order  $10^{-10}$  in the refractive index of left and right circularly polarized light in the medium.

### IV. RESULTS

Experiments were performed on a number of materials which are relatively easily available in a teaching laboratory. The materials investigated included distilled water, ethyl alcohol, methyl alcohol, BK7 glass, and aqueous solutions of ferric chloride at different concentrations. For liquid samples, a glass cuvette with 1-cm path length was used. The thickness of the BK7 glass used in our experiments was 1.2 cm. Lasers were used as light sources for the wavelength-dependent experiments simply because they were available, but a collimated white light source followed by an appropriate wavelength selective interference filter works fine. The wavelengths used in these experiments were obtained from an argon-ion laser (458, 476, 488, 496, and 514 nm), a diode pumped solid state laser (532 nm), a He–Ne laser (633 nm), and two diode lasers (670 and 830 nm).

Previously published Verdet constants of water at different wavelengths<sup>8</sup> were used as a standard to calibrate the magnetic field and check the results obtained with the gaussmeter. The rotation of the plane of polarization in distilled water as a function of the current passing through the coils was measured first and compared with previously published data to obtain an independent calibration of the magnetic field as a function of current. The wavelength-dependent results for distilled water are shown in Fig. 2(a) along with the published data from Ref. 8. The Verdet constants of ethyl alcohol, methyl alcohol, and BK7 glass were also measured and are shown in Fig. 2 (b), (c), and (d), respectively. The contribution from the glass cuvette has been subtracted for all liquid samples. The experimental error for the Verdet con-

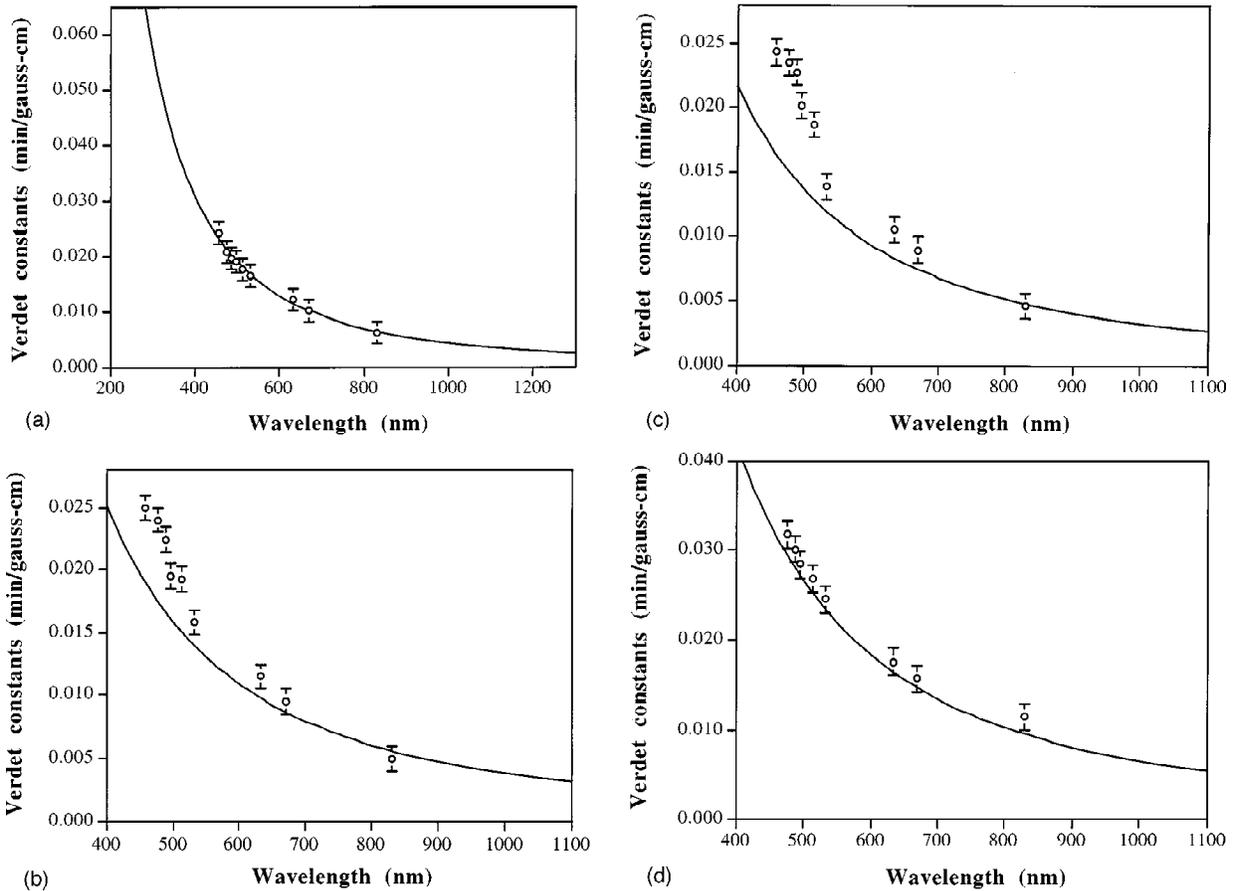


Fig. 2. The Verdet constants measured at various wavelengths along with previously published data of (a) distilled water, (b) ethyl alcohol, (c) methyl alcohol, and (d) BK7 glass, respectively. The solid lines present the power fit to previously published data and the open circles are experimental results.

stants determined from the experiment was estimated to be less than  $10^{-3}$  min/G cm and is mainly due to the leakage of the ac magnetic field from the coils and its pickup by the photodetector. This effect is minimized by enclosing the coils in a box made of iron or moving the detector further away from the region of the ac magnetic field. It is clearly seen from Fig. 2(a) and (d) that the experimental results of distilled water and BK7 glass are in good agreement with previously published data. However, for ethyl and methyl alcohols, deviations between the experimental data [Fig. 2(b) and (c)] and the published results are observed at short wavelengths. The data are consistent with the  $\lambda^{-2}$  dependence<sup>9</sup> of the Verdet constant for diamagnetic materials in the nonabsorbing wavelength regime.

Investigation of the concentration dependence of the Verdet constant has been carried out for aqueous ferric chloride solutions at 670 nm. The behavior of the Verdet constant is shown in Fig. 3. Our experimental data on water in Fig. 1 yield a positive value for the Verdet constant at 670 nm. In Fig. 3 we observe that the Faraday rotation changes sign from positive to negative values with increasing concentration of ferric ions. The ferric chloride solution at any particular concentration has a positive contribution to the Verdet constant from the water molecules. The ferric ions, however, are paramagnetic and make a negative contribution to the Verdet constant.<sup>9</sup> Consequently, as the concentration of ferric ion increases, the Faraday rotation decreases, and at large enough concentrations becomes negative. It is interesting to

note that solutions of paramagnetic salts of appropriate concentration can provide a medium with zero or exceedingly small Faraday rotations.

## V. CONCLUSIONS

An experiment suitable for a senior and graduate laboratory to measure the Verdet constant using inexpensive and

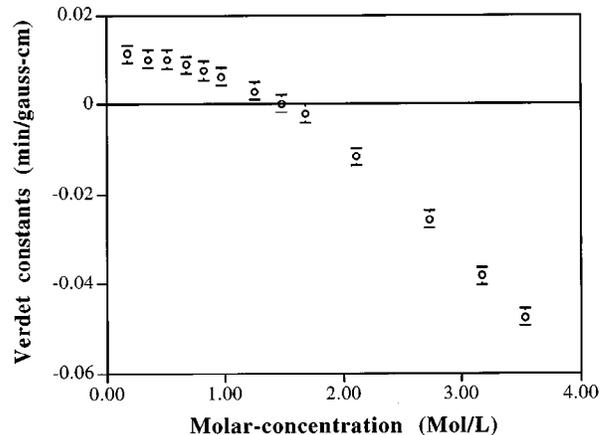


Fig. 3. Molar-concentration dependence of the Verdet constants for ferric chloride water solution measured at 670 nm.

readily available components has been developed. The Verdet constants of several commonly available materials as functions of wavelength were measured. Other than measuring Faraday rotation, this simple experiment can also be easily adapted for measuring other magneto-optic effects such as magnetic circular dichroism and the magneto-optic Kerr effect, and magnetic-field-induced changes in the gain of a cw diode laser.

## ACKNOWLEDGMENTS

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## PHYSICS AND POETRY

Why did it take Bohm so long to work out all the implications of his theory? Here a world of explanation is necessary, for although it may seem unusual to the nonscientist, the creator of a new theory may not necessarily fully understand what he or she has done.

A scientific theory may be compared to a deeply satisfying poem. Such a poem can be visited again and again during one's life. At each reading its density, allusions, images, and resonances are unfolded. The best poetry is an inexhaustible inscape, and its metaphors suggest a variety of different readings, none of which brings closure to its range of potential meanings. In similar fashion a scientific theory exists partly as mathematical formulae and partly as a cloud of words surrounding the formulae and their variables. Like the poem, the theory's meaning, or the way it functions, can be unfolded in a variety of different ways.

F. David Peat, *Infinite Potential—The Life and Times of David Bohm* (Addison-Wesley, Reading, Massachusetts, 1997), p. 14.