

# Non-linear dynamics with an $RL$ -Diode Circuit

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Have you ever wondered how science can be so organized and streamlined and yet be able to explain almost everything that we encounter? The answer to this genius skepticism is quite straightforward: we made it simple, for it was meant to simplify and systematize what seemed to be erratic and complicated. Science is a step towards intellectual sophistication in order to make things simple and explicable. This is why Leonardo da Vinci says, “simplicity is the ultimate sophistication.” In this experiment, we pursue an apparently simple example that can exhibit beautifully rich non-linear dynamics.

## **KEYWORDS**

Dynamical System · Phase Portrait · Poincare Map · Feigenbaum Constant · Diode Recovery Time · Junction Capacitance · Resonance · Period Doubling Bifurcation · Chaos.

**APPROXIMATE PERFORMANCE TIME** 1 week.

## **1 Objectives**

In this experiment we will discover:

1. how very simple systems can exhibit complex behavior under certain conditions,
2. the richness of the mathematical and physical structure of dynamical systems,
3. how an arbitrarily small change in the input can change the long-term conduct of a dynamical system drastically,

4. how to construct and interpret phase portraits and Poincare Maps for different kinds of responses of a system, and
5. the Feigenbaum constant and what makes chaos a universal underlying structure of the complexity exhibited by nonlinear dynamical systems.

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## 2 Foundations

**Summon up:** What kind of nonlinear phenomena have you come across? Try to list a few, with a reason to why you believe them to be nonlinear.

### 2.1 Defining Nonlinear Dynamics

Nonlinear systems are dynamical systems for which the principle of superposition does not hold are termed as nonlinear. For such systems the sum of responses to several inputs cannot be treated as a

single response to the sum of those all inputs. Qualitatively speaking [2]:

*A nonlinear system is a system whose time evolution equations are nonlinear; that is, the dynamical variables describing the properties of the system (for example, position, velocity, acceleration, pressure, etc.) appear in the equation in a nonlinear form.*

Now, if  $x$  represents an input variable and  $y$  is the output as a function of  $x$ , **the principle of superposition** in its very simplistic form states that:

$$y(x_1 + x_2 + \dots + x_n) = y(x_1) + y(x_2) + \dots + y(x_n) \quad (1)$$

The above mathematical expression means that if the stimulus to a linear system is doubled, the response is also doubled. For a nonlinear system, the response will be greater or less than that.

What makes nonlinearity so important? The basic idea is that for a linear system, when a parameter (e.g. the spring constant  $k$  in a spring mass system) is varied, it doesn't change the *qualitative* behavior of the system. On the other hand, for nonlinear systems, a small change in a parameter can lead to sudden and dramatic changes in both the qualitative and quantitative behavior of the system. For one value, the behavior might be periodic. For another value only slightly different from the first, the behavior might be completely aperiodic.

## 2.2 Chaos

In the context of nonlinear dynamical systems, chaos is a word used to describe the time behavior of a system that is aperiodic, and is *apparently* random or "noisy". But, underlying this chaotic randomness is an order that can be determined by the time evolution equations that describe the system. Even when it may sound paradoxical, such an apparently random system is in fact deterministic. Chaos is identified by hypersensitive dependence on initial conditions. A minute change in initial conditions can lead to drastically differing dynamical evolution of the system.

Generally, chaos starts with a so called *period-doubling bifurcation*: system switches to a new behavior with twice the period of the original system at a particular value of a certain parameter. As the value of that parameter is further increased, successive bifurcations occur and the behavior of system takes a time period that is four times, then eight times and so on, finally ending up in chaotic behavior.

**The math ingredient:** A dynamical system is expressed by its differential equations. Consider what happens to the solution of the system equations when a bifurcation occurs?

## 2.3 Phase portraits

The notion of state space (or phase space) is a very rich topic. It helps in the quantitative inspection of dynamical systems. The phase portrait is a trajectory of the system, shown in time, with two conjugate variables plotted against each other. For example, for a periodic system exhibiting simple harmonic motion (e.g., a pendulum), the phase portrait will be a closed loop for a particular set of initial conditions. The conjugate variables for the pendulum are the angle  $\theta$  and its derivative,  $\dot{\theta}$ , which is proportional to the angular momentum. For a chaotic system, there will be many distinct loops in a phase portrait, showing that the system is aperiodic and does not approach a stable trajectory.

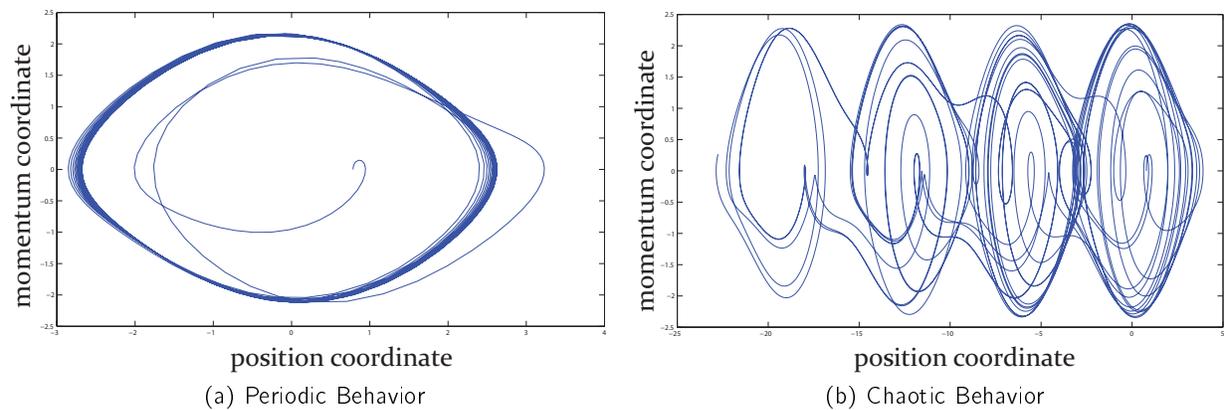


Figure 1: Possible phase portraits of periodic and chaotic behavior. Refer to main text for labeling of axis.

## 2.4 Example of a simple pendulum

Consider a simple pendulum as shown in Figure 2 having a small amplitude of oscillation (so that we can assume  $\sin \theta \approx \theta$ ). Ignoring friction, it may be represented using Newton's second law by a normalized second order differential equation of the form:

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\theta = 0 \quad (2)$$

where  $\theta$  represents the angular position of the pendulum. The solution of this equation is:

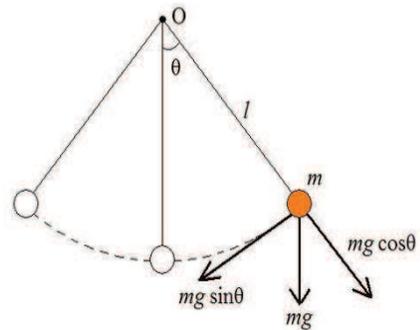


Figure 2: The simple pendulum.

$$\theta = \theta_o \sin(\omega t + \phi) \quad (3)$$

where  $\theta_o$  is the maximum angular displacement. The first derivative of  $\theta$  is:

$$\dot{\theta} = \theta_o \omega \cos(\omega t + \phi) \quad (4)$$

Now, from (3) and (4), writing an equation in terms of  $\theta$  and  $\dot{\theta}$  will give the parametric equation:

$$\frac{\theta^2}{\theta_o^2} + \frac{(\dot{\theta})^2}{(\omega\theta_o)^2} = 1 \quad (5)$$

which is evidently the equation of an ellipse with  $\theta$  on the horizontal and  $\dot{\theta}$  on the vertical axis and represents a periodic trajectory in the phase space. In this context,  $\theta$  and  $\dot{\theta}$  represent the canonical coordinates.

**Implicate:** Write down the equation of energy of a pendulum in terms of position and momentum variables indicated in the formalism above. What is the total energy in the system?

**Figure out:** What does a closed loop in phase space signify? What can we say about the energy contained in a system?

**A step ahead:** Draw the circuit diagram of an *RLC* circuit. Write down the differential equation of the system and identify the canonical coordinates.

## 2.5 Poincare sections

Another very useful way of analyzing the behavior of a nonlinear dynamical system is a Poincare Section or Poincare Map. The basic motivation behind making such a map is to reduce an  $n$ -dimensional system to an  $(n - 1)$ -dimensional system, making the analysis easier.

Constructing a Poincare map is simple: sample the phase portrait of the system stroboscopically [7]. For periodic behavior, the Poincare map will be a single point or a well-identified distinct groups of points. For chaotic or aperiodic behavior, there will be many irregularly distributed points in the map.

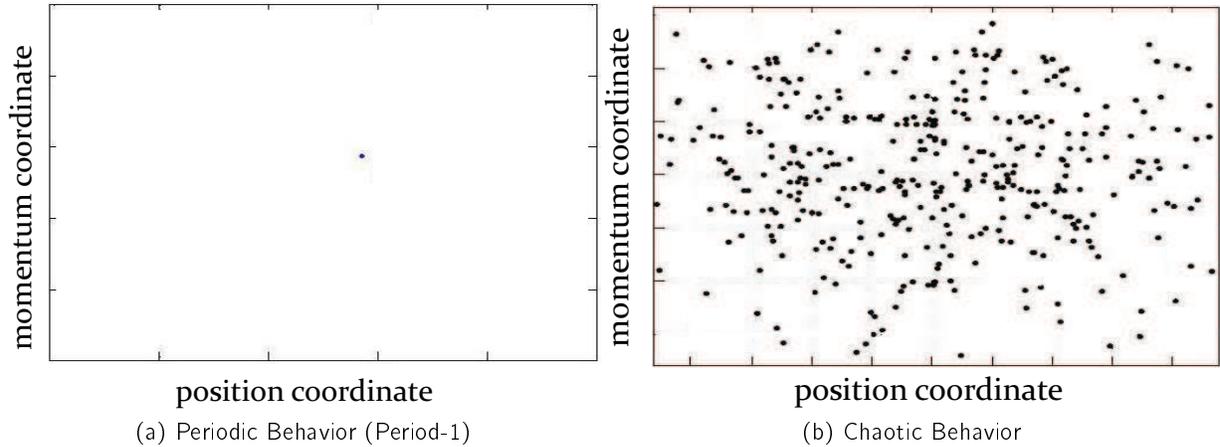


Figure 3: Poincare sections for periodic and chaotic behavior.

## 2.6 Bifurcation diagrams

Yet another method of expressing the behavior of a dynamical system over the entire range of a particular parameter is the bifurcation diagram. It shows a correspondence between the parameter values and the resulting response of the system. Every bifurcation indicates a successive period doubling and the response branches off into two. For example, in figure (4), as the control parameter  $\lambda$  is varied over a certain range, the response  $x_n$  takes different number of values: two values at the first bifurcation, four values at the second bifurcation, eight values at the third bifurcation and so on. The fuzzy bands indicate chaotic behavior. Also, one can observe the periodic bands within the chaotic ones, showing that chaos can suddenly vanish and give rise to certain higher order periods. This is mainly because of the fact that differential equations defining the system may abruptly switch from chaos to a definite set of solutions for a certain value of the control parameter.

## 2.7 Universality of chaos and the Feigenbaum Constant

When we look at a bifurcation diagram, such as the one shown in figure (4), we can see the distances between successive bifurcations getting smaller and smaller in a geometric way (along the horizontal axis). This is what Feigenbaum noticed: *the ratio of differences of parameter values at which successive bifurcations occur is the same for all the splittings* [2].

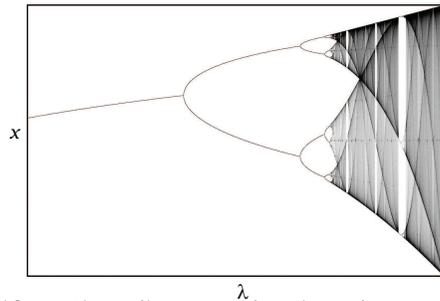


Figure 4: Bifurcation diagram showing the response  $x$  as a function of the control parameter  $\lambda$ . (source: wikipedia.org)

Mathematically speaking:

$$\delta_n = \frac{\lambda_n - \lambda_{n-1}}{\lambda_{n+1} - \lambda_n} \quad (6)$$

where  $\lambda_n$  is the parameter value at which the  $n$ 'th bifurcation occurs. Moreover, this ratio converges to a particular value called the Feigenbaum constant as  $n$  approaches infinity:

$$\delta \equiv \lim_{n \rightarrow \infty} \delta_n = 4.669201 \dots \quad (7)$$

### 2.7.1 Attractors and fractals

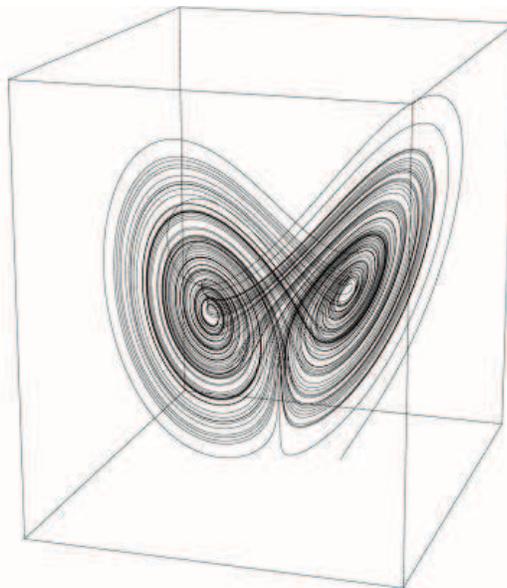


Figure 5: The Lorenz attractor: state-space trajectories are confined.

An important manifestation of the fact that chaos is deterministic are *attractors*: a set of points (or trajectories) to which all other trajectories—that start from the initial conditions lying within a region called the *basin of attraction*—approach, as time goes to infinity. Looking at the accompanying figure, we can observe how trajectories remain within a certain region of state-space. This *confinement* of trajectories within a certain region for a particular set of initial conditions is what points toward the determinism in the chaotic behavior.

Attractors, in addition to their aesthetic appeal and tendency to provide us with information about the active degrees of freedom in a system, also determine the dynamical properties of the system's long-term behavior.

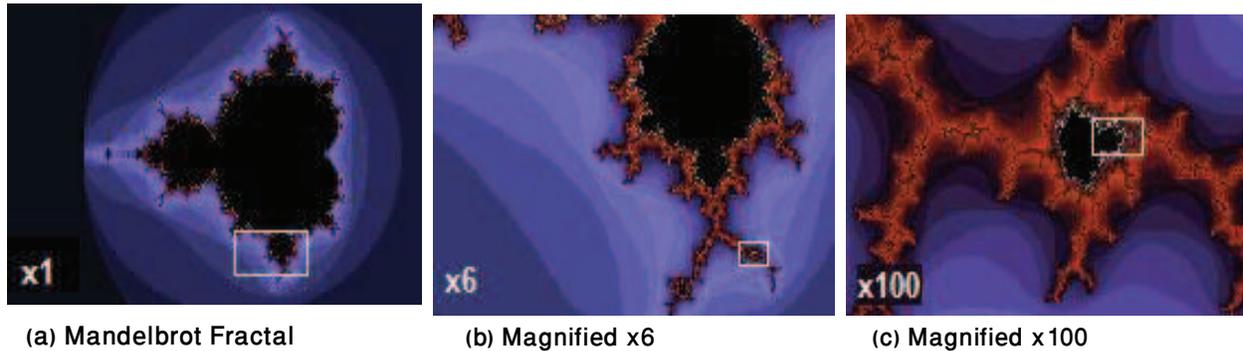


Figure 6: Mandelbrot fractal: regions indicated in boxes are magnified. A resemblance with unscaled image can be noticed even when magnified 100 times.

The discussion on attractors cannot go without mentioning one of the most aesthetically rich notions in mathematics, namely fractals, that actually link attractors with the universality of chaos. Fractals are self similar and self replicating geometrical structures (figure (8)) that occur in the state space as attractors with a *noninteger dimension* and are sometimes called *strange attractors*. Noninteger dimension refers to the idea that, in general, these geometrical figures do not have a point, axis or plane of symmetry, and yet they are self-similar within themselves: they look the same at any degree of magnification.

### 3 The Experiment

The subject of this experiment is a simple RL-Diode circuit. Although simple it may seem, yet it exhibits interesting behavior including bifurcations and chaos. A series arrangement will be used as shown in the accompanying figure.

#### 3.1 The Circuit

The circuit (figure (7)) will alternately behave in two different modes when subject to an AC voltage source: first when the diode is forward biased, the other when it is reverse biased.

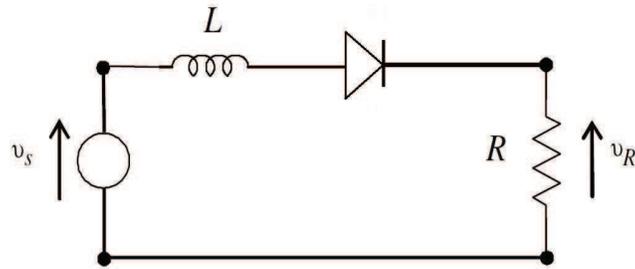


Figure 7: The Experimental RL-Diode circuit.

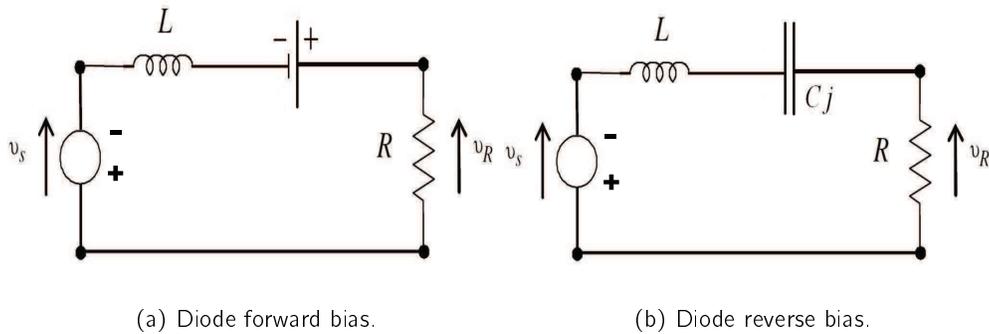


Figure 8: Equivalent circuits for forward and reverse bias cycle.

### 3.2 The Mathematical Model

**During the conducting cycle**, the circuit reduces to what is shown in figure (8a), with the diode acting as a fixed voltage drop, i.e., a battery. The Kirchoffs voltage law applied to this circuit gives:

$$L \frac{di}{dt} + Ri = V_o \sin \omega t + V_f \tag{8}$$

where  $V_o$  is the peak amplitude of the AC input voltage and  $V_f$  is the diode's forward voltage drop, which is generally about 0.5-1.0 V. The solution of this equation, i.e., the current in the conducting cycle is easily found [4]:

$$i(t) = \left( \frac{V_o}{Z_a} \right) \cos(\omega t - \theta) + \frac{V_f}{R} + Ae^{-Rt/L} \tag{9}$$

In the above equation  $\theta = \tan^{-1}(-\omega L/R)$  represents a phase delay;  $A$  is a constant of integration to be calculated using the initial conditions and  $Z_a = \sqrt{R^2 + \omega^2 L^2}$  is the forward bias impedance of the

circuit .

**In the non-conducting cycle**, the diode behaves as a capacitor having a capacitance equal to its junction capacitance ( $C_j$ ). The equivalent circuit can be represented as a driven  $RLC$  circuit (figure (8b)). The loop equation becomes a second order differential equation:

$$L \frac{d^2 I}{dt^2} + R \frac{dI}{dt} + \left( \frac{1}{C_j} \right) I = V_o \omega \cos \omega t \quad (10)$$

**Derive:** Derive the above equation from Kirchoff's voltage law.

Equation (10) can be solved using traditional techniques.

**Derive:** Derive the solutions of Equations (8) and (10). The solution for Equation (8) is given in (9) while the final solution of equation (10) is given below:

$$I(t) = \left( \frac{V_o}{Z_b} \right) \cos(\omega t - \theta_b) + B e^{-Rt/2L} \cos(\omega_b t - \phi) \quad (11)$$

The constants  $B$  and  $\phi$  are constants of integration and can be found using the initial conditions of the cycle. Moreover,  $\theta_b = \tan^{-1}(\omega R/L(\omega_o^2 - \omega^2))$ ,  $\omega_o^2 = (1/LC_j)$ ,  $\omega_b^2 = \omega_o^2 - (R/2L)^2$  and  $Z_b = (\frac{L}{\omega}) \sqrt{(\omega_o^2 - \omega^2)^2 + (\frac{R\omega}{L})^2}$

### 3.3 The Physical Model

#### 3.3.1 The diode recovery-time

Prior to looking into the practical behavior of the circuit and how it becomes chaotic, we need to understand the meanings and significance of an important parameter: the diode's recovery time. The recovery time of a diode is the time a diode would take to completely stop the flow of forward current through itself as it moves into the non-conducting cycle. It depends on the amount of maximum forward current that has just flown through the diode. The greater the peak forward current, the longer the diode recovery time. Quantitatively speaking the recovery time is given by [4]:

$$\tau_r = \tau_m [1 - \exp(-|I_m|/I_c)] \quad (12)$$

where  $|I_m|$  is the magnitude of the recent most maximum forward current, and  $\tau_m$  and  $I_c$  are fabrication parameters for the specific diode.

**Bring to Light:** What is the physical explanation of a diode's junction capacitance? What relationship does it have with the recovery time?

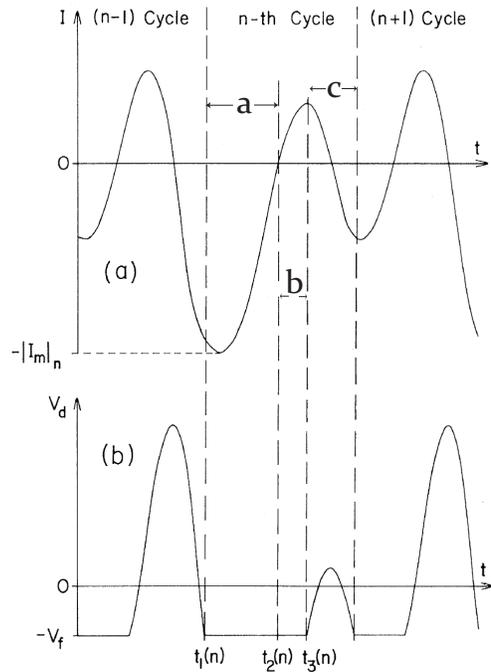


Figure 9: Circuit current,  $I$ , and diode voltage,  $V_d$ , (period-2) [4]. The diode conducts when  $V_d = -V_f$  behaving as in the circuit in figure (10a). Otherwise it behaves as a capacitor as shown in figure (10b).

### 3.3.2 The period-doubling route to chaos

A physical description of how the  $RL$ -diode circuit leads to period doubling is described in detail in [4]. Here we recapitulate the most important points.

A certain amount of reverse current will flow through the diode in every reverse bias cycle due to the finite recovery time of the diode. If the peak current  $|I_m|$  is large in the conducting cycle (figure (9), interval 'a'), the diode will switch off with a certain delay (figure (9), interval 'b') due to the finite recovery time and so will allow a current to flow even in the reverse-bias cycle (shown in the interval 'b'). This reverse current, in turn, will prevent the diode from instantly switching on in the forward bias cycle; it will turn *on* with a delay (figure (9), interval 'c'). This will keep the forward peak current smaller than in the previous forward bias cycle, hence giving birth to two distinct peaks of the forward current. Notice that it took *two* cycles of the driving signal in this process. This is what we identify as a period-doubling bifurcation.

When the peak value of the drive voltage is increased, bifurcation to period-4 may occur followed,

possibly, by higher bifurcations and eventually chaos.

**Self-Assessment:** Briefly explain figure (9) according to the labels on the time axis, describing what happens at every marked instant.

### 3.4 The Procedure

#### 3.4.1 The Setup

The list of equipment used in this experiment is listed here.

1. Oscilloscope (Agilent DSO-X 2002A)
2. Function Generator (BK Precision 4086)
3. Data Acquisition Setup (National Instruments DAQ card)
4. Spectrum Analyser (Agilent N9320B)
5. RL-Diode circuit components-  $15\ \Omega$  Resistor,  $\approx 7.04\ \text{mH}$  Inductor, 1N4007 Diode

Chaos and bifurcations are usually observed by only changing one parameter (and keeping all others constant) and observing the response of the system. In our case the system is an *RLD* circuit in which current and voltage will be made to oscillate using an AC signal. The signal is controlled through two parameters: frequency and amplitude. So we have the option of keeping one of them constant and changing the other one. In our case frequency will be kept constant while amplitude is varied.

It is now time to start our experimental expedition.

**Before you begin:** Keep the operating manuals of the oscilloscope and signal generator handy. You will have to frequently consult these.

### 3.4.2 Time-series of voltage across resistor.

Connect the circuit in series to the signal generator. Using a BNC-to-Crocodile clips cable, feed  $V_R$  into Channel 1 of the Agilent oscilloscope.

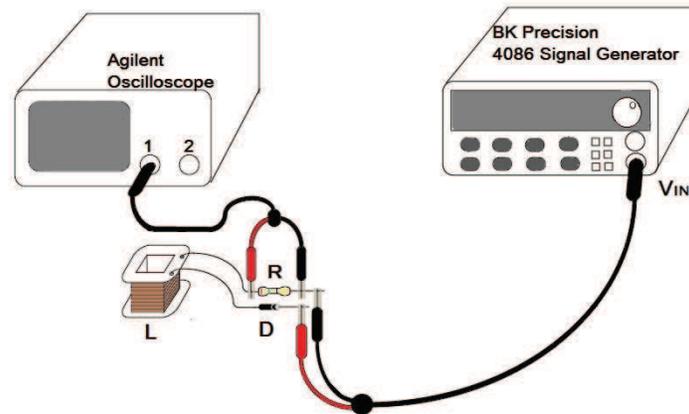


Figure 10: Schematic for the circuit for measuring  $V_R$  across the resistor.

Fix the frequency of  $V_{IN}$  using the BK4086 signal generator to 50 kHz. (**NOTE:** The frequency will be kept fixed for all future segments of the experiment.)

Now change the amplitude of  $V_{IN}$  in steps of 0.01 V and observe and record the changes in the  $V_R$  waveform on the oscilloscope. Keep adjusting your oscilloscope scales accordingly to view the full waveform. You expect to see a period-1 ( $\frac{1}{50}$  ms) waveform, which transitions to period-2, period-4 and so on (See sample results posted online).

When bifurcations occur you will also need to alter the trigger level (using the trigger knob) of the oscilloscope so that bifurcation peaks are clearly visible on the waveforms. A more stable and finer waveform can be formed by increasing the number of samples (of the input signal at Channel 1) being averaged out by the oscilloscope. The number of averages can be increased by pressing **Acquire** on the oscilloscope and increasing the averaged number to 128.

#### Questions

1. When does the first bifurcation occur ( $2T$ )?
2. When does the second bifurcation occur ( $4T$ )?

3. Are you able to see the third bifurcation ( $8T$ )?  
 Sketch the waveform plots of  $V_R$  for  $1T$ ,  $2T$ ,  $4T$  and  $8T$  bifurcations and describe them in your notebooks.

**TIPS:**

1. The voltage values (of  $V_{IN}$ ) at the first point of *onset* of the bifurcation peaks should be set as the voltage values where the bifurcations take place.
2. Do not decrease the amplitude of  $V_{IN}$  while you are increasing the amplitude to observe bifurcations because voltage values at which bifurcations occur differ for cases where you increase or decrease the amplitude due to hysteresis in the system. Hysteresis is an important feature of nonlinear dynamical systems.

**3.4.3 Time-series of voltage across diode.**

Connect the circuit in the same way before except that in this case you will be feeding  $V_D$  to channel 1 of the oscilloscope. You will also have to reverse terminal connections inside the circuit as shown in the schematic below. Precisely following the color scheme below will help!

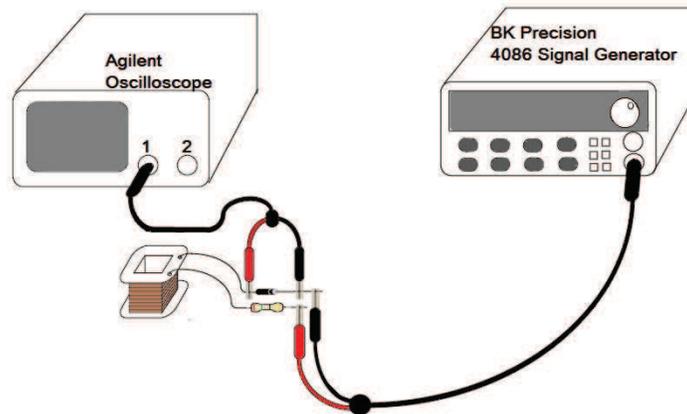


Figure 11: Schematic for measuring  $V_D$  across resistor.

Repeat the whole process as you did for waveform time-series of  $V_R$ . And note down the voltage values at which bifurcations occur.

**Question** How do your voltage values compare for bifurcations compare for  $V_R$  and  $V_D$ ?

A diode has nearly a finite potential drop,  $V_R$ , when its forward biased (in the case of 1N4007 it is 1 V). In the reverse bias mode the diode does not conduct (although it does become conducting

after the point of blocking voltage has been passed). Now we are going to introduce a DC offset in our  $V_{IN}$  and study the effects of it on  $V_D$  waveform visible on the oscilloscope.

Select a voltage value now where you can clearly see a stable second bifurcation ( $2T$ ) on the waveform. Now introduce a DC offset into your  $V_{IN}$ . First introduce an offset of +1 V and then an offset of -1 V into your signal.

To introduce an offset using the signal generator press **Shift** and then press **Offset/Gate** and using the knob select your amplitude for the offset.

**Question** What happens to the peaks in the case of +1 V offset? In the case of -1 V? Record and explain your observations.

#### 3.4.4 Observing period bifurcations on the spectrum analyzer and determining the Feigenbaum constant.

As it is very hard to determine and clearly observe the point at which the third bifurcation occurs we will now use a spectrum analyzer. It is much easier to identify the third bifurcation by seeing the Fourier spectrum of the signal. Therefore the spectrum analyzer will help us in identifying the different frequencies present in the signal by creating the spectral density graph and help identify the voltage values for the respective bifurcations.

**Calculate** Use Equation (6) and voltage values for  $2T$  and  $4T$  from previous sections to calculate and predict the voltage at which the third bifurcation should occur ( $8T$ ).

The amplitude will again be varied manually in steps of 0.01 V using the signal generator but this time the signal  $V_R$  (across the resistor) will be fed into the spectrum analyser.

To set up the spectrum analyser follow the following steps:

1. After you have turned the analyser on press **Preset/System**.
2. Now press **Preset**. This will take the analyser to its factory settings.
3. To calibrate the analyser first connect a BNC cable between **CAL OUT** and **RF IN**.
4. Then press the buttons in the following sequence **Preset/System** → **Alignment** → **Align** → **All**.  
The system will take a minute to align itself.

Here are a few tips for proficiently using the spectrum analyzer.

1. Now connect the BNC which gives the voltage output across the resistor ( $V_R$ ) and connect it to **RF IN**. Connect all the circuit components in the same way as shown in the schematic below.

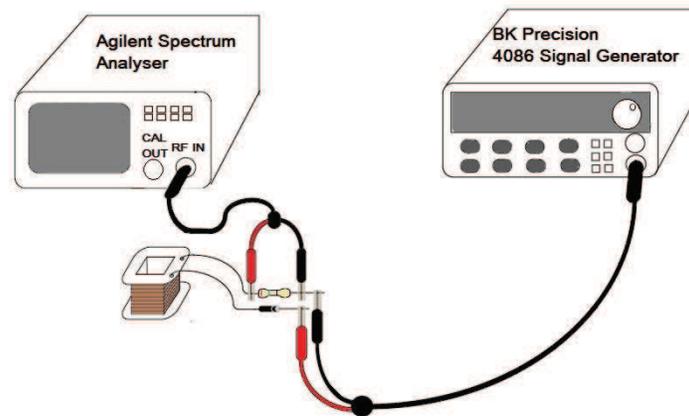


Figure 12: Connecting the circuit to the spectrum analyser.

2. To change the width of the spectral window press **Frequency**. Then press **Stop Frequency** and **Start Frequency** to set frequency limits of the window.
3. To make the peaks clearer and more distinguishable press **Amplitude**. Then press **Scale Type** and press it again to select **Lin**. Now press **Ref Level** and turn the knob to adjust the reference level.
4. To change the band width resolution press **BW/Avg** and all the options will be in front of you.
5. Set the number of **Average** samples to be 30 and set the **Average type** to power.
6. You can check the amplitude and the frequency corresponding to each peak by **Peak Search** and then switching between the peaks. Or if you want to go to a certain point on the spectrum press **Marker** and then rotate the knob to move the marker around the spectrum.

**Explain** The peaks which arise when bifurcations occur always arise to the left side of the main signal peak (50 kHz). Where do the peaks at the right of the main peak come from?

**Observe** Using the spectrum analyser you have to note down the value of voltages at which the three bifurcations occur. Note down the frequency and the amplitude of each of the spectral peaks for each of the bifurcations.

### HINTS & TIPS

For period one ( $1T$ ) you will see one peak in the spectrum. For period two ( $2T$ ) you will see two peaks one at 50 kHz and one at  $\frac{50+0}{2} = 25$  kHz. For period four ( $4T$ ) you will see four peaks and their respective frequencies will be 50,  $\frac{50+25}{2} = 37.5, 25$  and  $\frac{25+0}{2} = 12.5$  (all in kHz).

**Calculate** What will be the frequency values of each of the 8 peaks you expect to see in period eight?

Each time you may have to change the reference level, the spectral window and resolution of the band width to be able to see the peaks clearly. For example while trying to observe the point of onset of the 25 kHz period two peak you can have a reference level of  $112.1 \mu\text{V}$ , a spectral window of 24 to 26 kHz and a band with resolution of 30 Hz. For period 8 peaks you will have to decrease the reference level and increase the band width resolution.

**Question** What is your value for the Feigenbaum constant? Do your voltage values for the bifurcations taken using the oscilloscope and the spectrum analyser match?

### 3.4.5 Plotting phase portraits.

Connect the circuit as shown in the schematic diagram below:

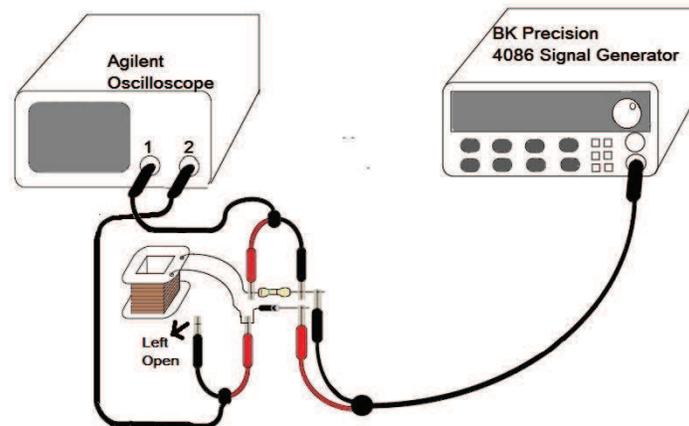


Figure 13: Schematic for the circuit for observing phase portraits on the oscilloscope.

1. Turn on the oscilloscope and the signal generator.
2. Bring the oscilloscope into XY mode. On the oscilloscope you will now be observing the phase portrait of this RLD circuit. Channel 2 shows a signal proportional to the current  $I$ , and channel 1 is connected to a signal proportional to  $\frac{dI}{dt}$ , which are the variables for this system.
3. Start increasing the frequency from 0  $V_{p-p}$  and keep increasing it to 20  $V_{p-p}$  in steps 0.01 V.
4. Observe the changes in the phase portrait. Keep adjusting your oscilloscope scales accordingly to view the full phase portrait clearly.

## Question

1. When does the first bifurcation occur ( $2T$ )?
2. When does the second bifurcation occur ( $4T$ )?
3. Can you see period eight? Sketch and record all your phase portraits on your notebook.

### 3.4.6 Plotting the bifurcation diagrams using an amplitude sweep.

In this final step, we will make a bifurcation diagram. For this purpose, we will automatically sweep the input voltage ( $V_{IN}$ ) in a linear fashion. Using the NI DAQ card single up-ramp voltage pulse will be generated. This will be used to modulate the sine wave being produced by the function generator (BK 4086). In order to plot the bifurcation diagram, we need to pick up the peak (or valley) values from a time series of, say, the voltage across the resistor  $V_R$ . The scheme is depicted in the flowchart (Figure 14). Make sure you understand how the process works.

Perform the following steps in order to configure the signal generator (BK 4086) enabling it to perform amplitude modulation :

1. Press **[AM]** to activate AM mode.
2. Press **[frequency]**, then **[50] [kHz]** to set the frequency of the carrier wave. This can also be done by turning the knob
3. Press **[amplitude]**, then **[2] [0] [V]** to set the amplitude of the carrier signal. This can also be done by turning the knob.
4. Now when you press **[menu]** **AM LEVEL** will appear on the screen. To set the **AM LEVEL** press **[1] [0] [0] [N]**.
5. Press **[menu]** again and now **AM FREQ** will appear on the screen and you have to select the frequency/timeperiod of the increasing-ramp wave. As we will be using an external source (DAQ card) for modulation therefore no specific value has to be set for **AM FREQ**.
6. The same will be the case for **AM WAVE**.
7. Lastly press **[menu]** to select the **AM SOURCE** and set it to external by pressing **[2] [N]**.
8. Now to finish off the process press **[menu]** again.

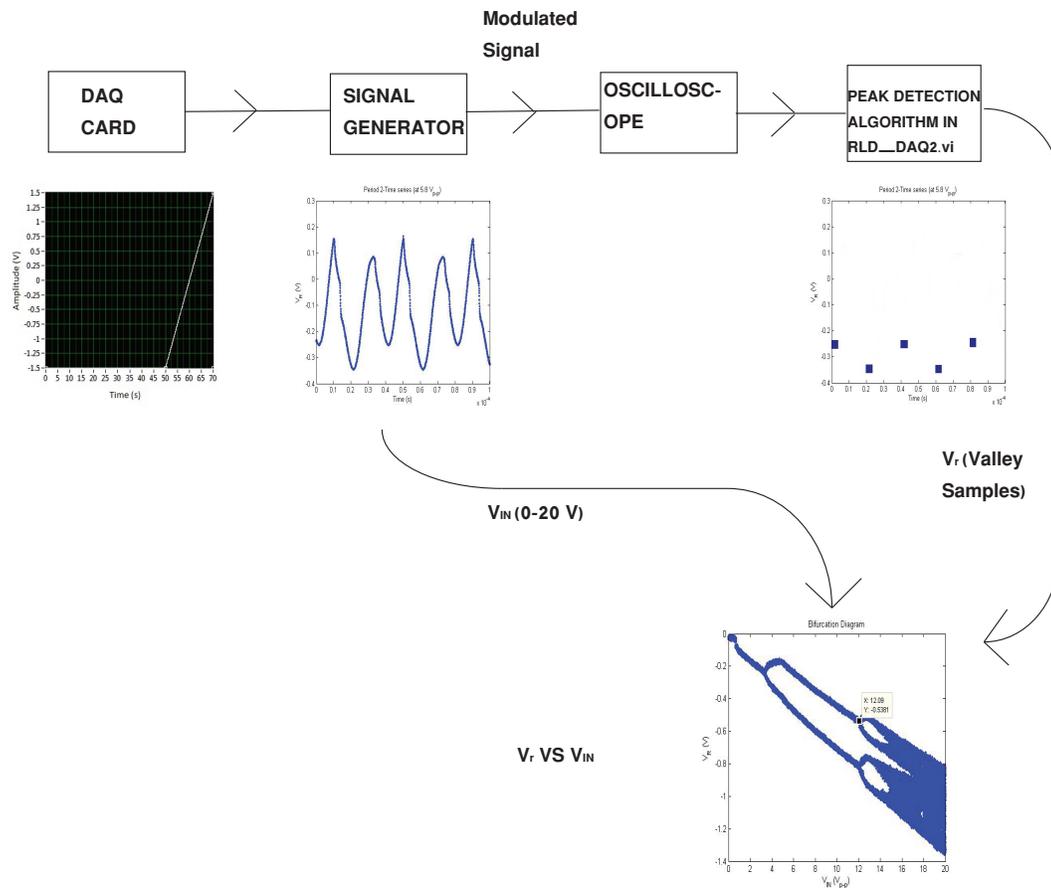


Figure 14: Schematic for the circuit for plotting bifurcation diagram using oscilloscope, signal generator and a data acquisition card's output channel controlled using LabVIEW.

Now the signal generator will generate a sinusoidal pulse (50 kHz, 10  $V_{pp}$ ) whose amplitude will be modulated by an increasing-ramp pulse produced by the DAQ card. The ramp pulse will have a period of 20 s and will change the amplitude of the sine wave from 0 to 20  $V_{pp}$ .

Connect the RLD circuit with the oscilloscope as you did for in the case of a phase portrait in the previous section. In this case you will also need to connect the DAQ card output terminals to the BNC input terminal (for external modulating signal) at the back of BK 4086. You will need a BNC-to-crocodile clips cable. The BNC cable will be attached to the **MOD IN (3 V= 100 %)** terminal on BK 4086. The black clip of the cable will be attached to the orange (-1.5 V) DAQ terminal wire and the red clip will be attached to the green (+1.5 V) DAQ terminal wire.

Turn on your computer and open the **RLD\_DAQ2.vi** file. The interface would look something similar

to what is shown in Figure 15.

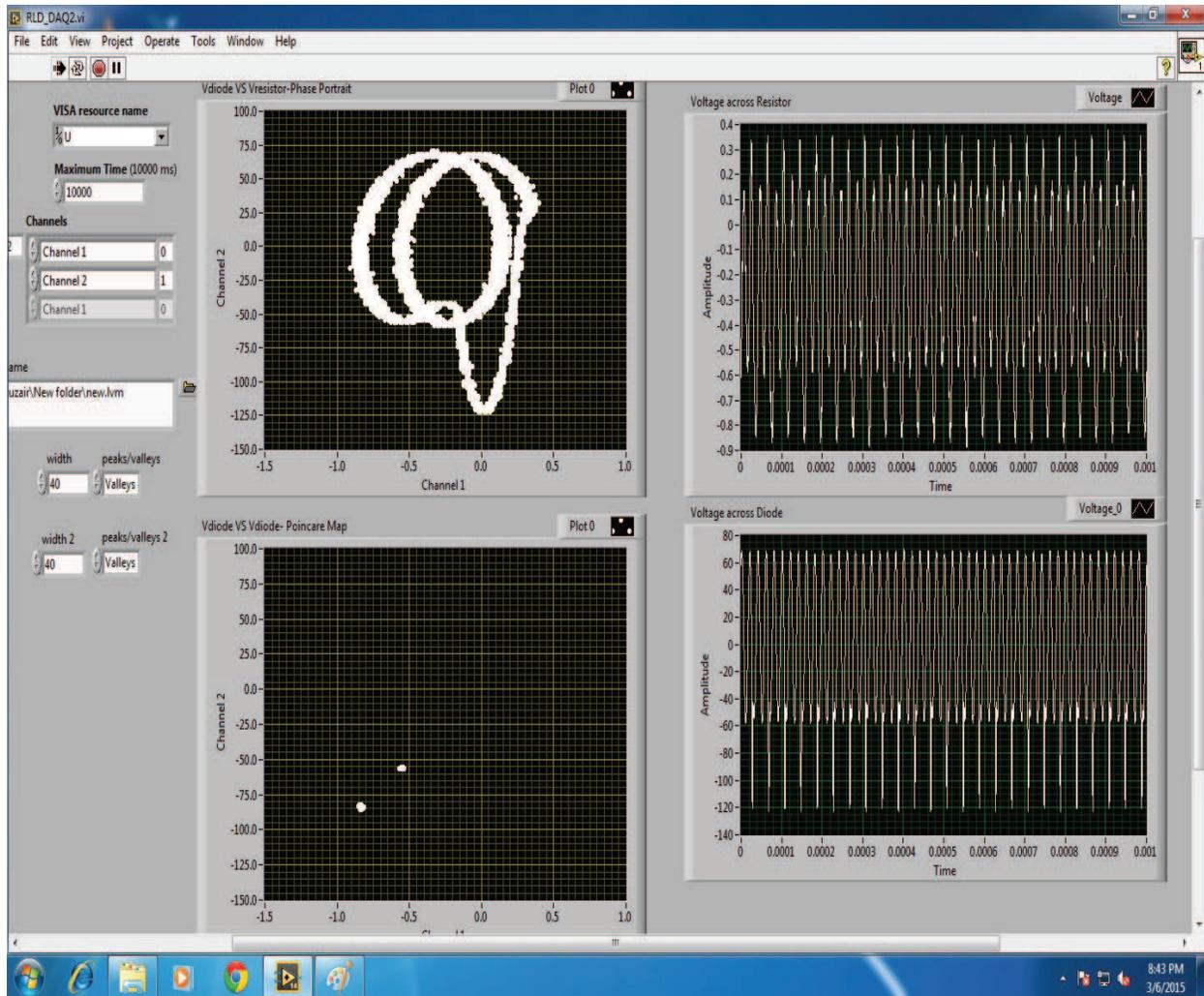


Figure 15: The RLD\_DAQ.vi file interface on the computer.

In the interface you can also see the phase portrait and the Poincare map graphs of the signals from the RLD circuit being fed into the computer.

- Set the **width** to **40** and set the **Peaks/Valleys** to **Valleys**.

When you will run the VI it will take around 15 s for the oscilloscope to start feeding live data to the computer. When the vi has started running you will have to adjust the scales on the oscilloscope ASAP.

**IMPORTANT:** These adjustments would have to be done using the oscilloscope knobs before 47 s

mark on the VI clock. At the 47 s mark our modulated pulse will be initiated.

1. Set the Channel 1 amplitude scale to 500 mV
2. Set the Channel 2 amplitude scale to 50 V.
3. Set the horizontal time scale to 5 MSa/s.

Bifurcation diagrams like the one shown in Figure 4 are made by sampling peak amplitudes of the AC voltage signal in the RLD circuit. In period-1 all the peaks have the same amplitude, in period-2 there are peaks of two different amplitudes, in period-4 there are peaks of 4 different amplitudes present and vice versa. So a parameter (in this case the amplitude,  $V_{IN}$ ) is varied over a period of time and the peaks are sampled. The sampled peak amplitudes of  $V_R$  are plotted against the corresponding parameter (amplitude) value. This gives us the bifurcation graph.

Start your VI. The VI would run for a total of 70 s.

As the VI would be running this sampled data would be being recorded in the data file which you would have specified on the VI interface. The data file would have 3 columns: the time column, the location column and the amplitude column. The amplitude column would contain values of all the valley amplitudes.

First load the data file in MATLAB in variable **b** in MATLAB. You will have to plot the amplitude column against  $V_{IN}$  whose values will range from 0 to 20 V. In order to create a vector of  $V_{IN}$  you will have to use the command in MATLAB of **linspace(0,20,N)** where  $N$  is the number of samples you have for the amplitudes of  $V_R$  which will be equal to the length of the **b** vector.

Plot column 3 of **b** against  $V_{IN}$  and you will get something similar to the following figure.

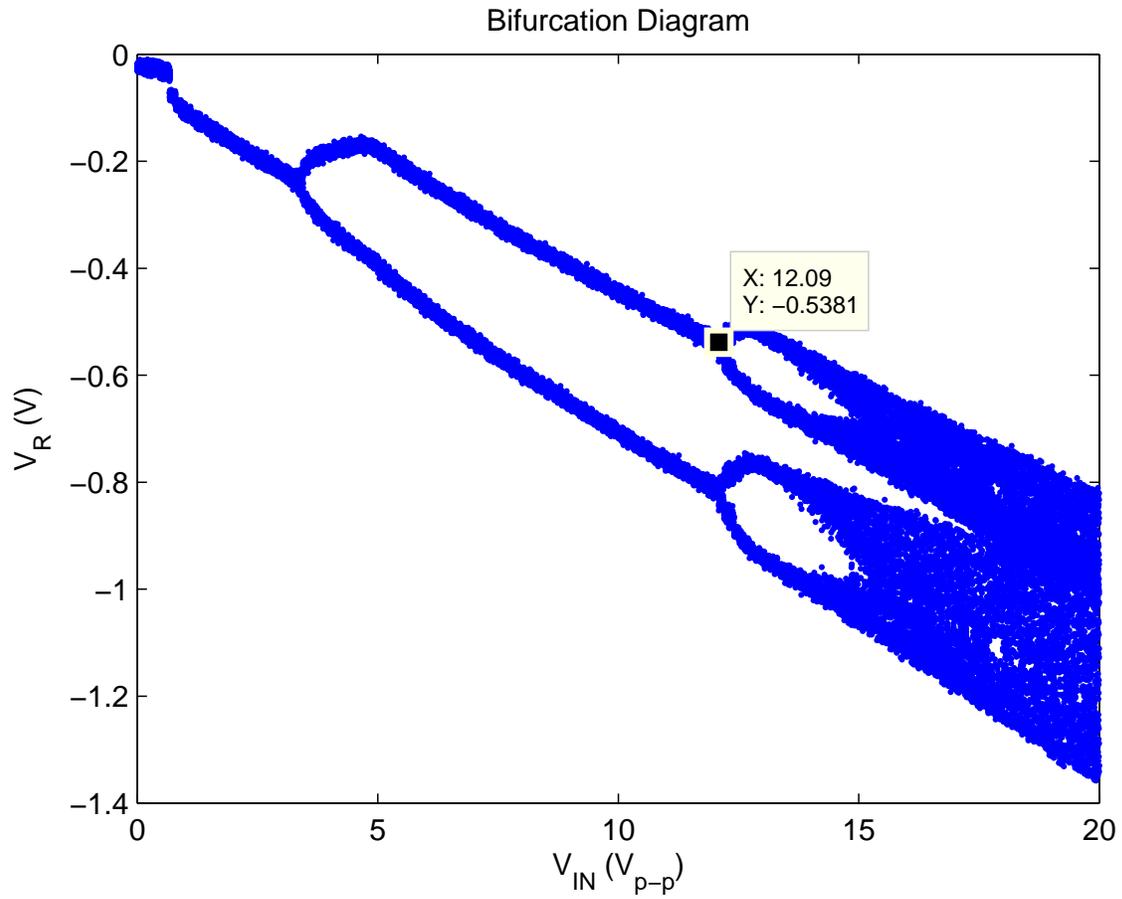


Figure 16: The raw bifurcation you would obtain after running your VI for about 70 s.