Non-linear dynamics with an RLD-Circuit

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This experiment is based on Chaos theory and the nonlinear dynamics of an RLD-circuit. By employing different data-acquisition techniques and representing them via time-series plot, phase portrait, Poincare section, spectrum density plot, and bifurcation diagram, we aim to observe the varying period of the voltage signal across the resistor. Each of these plots are an attempt to present a clear picture of bifurcation—the point where period-doubling takes place. We want to be able to distinguish higher order periods such as 8T from chaos using these plots.

KEYWORDS

Dynamical System · Phase Portrait · Poincare Map · Feigenbaum Constant · Diode Recovery Time · Junction Capacitance · Resonance · Period Doubling Bifurcation · Chaos.

APPROXIMATE PERFORMANCE TIME 1 week.

1 Objectives

In this experiment we will discover:

1. how very simple systems can exhibit complex behavior under certain conditions,
2. the richness of the mathematical and physical structure of dynamical systems,
3. how an arbitrarily small change in the input can change the long-term conduct of a dynamical system drastically,
4. how to construct and interpret phase portraits, bifurcation diagrams, and Poincare Maps for different kinds of responses of a system.

5. the Feigenbaum constant and what makes chaos a universal underlying structure of the complexity exhibited by nonlinear dynamical systems.

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2 Foundations

Summon up: What kind of nonlinear phenomena have you come across? Try to list a few, with a reason to why you believe them to be nonlinear.

2.1 Defining Nonlinear Dynamics

Nonlinear systems are those dynamical systems for which the principle of superposition does not hold. For such systems, the sum of responses to several inputs cannot be treated as a single response to the sum of those all inputs. In other words, for such systems, the change in the output parameter is not proportional to that of the input parameter. On increasing the input linearly, the corresponding output variable shows different behaviours—ranging from doubling of the output to four times to chaos and then back to an integer multiple of the initial value (it could be some other pattern too).

Now, if x represents an input variable and y is the output as a function of x, **the principle of superposition** in its very simplistic form states that:

$$y(x_1 + x_2 + ... + x_n) = y(x_1) + y(x_2) + ... + y(x_n) \tag{1}$$

The above mathematical expression means that if the stimulus to a linear system is doubled, the response is also doubled. For a nonlinear system, the response will be greater or less than that.

What makes nonlinearity so important? The basic idea is that for a linear system, when a parameter (e.g. the spring constant k in a spring mass system) is varied, it doesn't change the *qualitative* behavior of the system. On the other hand, for nonlinear systems, a small change in a parameter can lead to sudden and dramatic changes in both the qualitative and quantitative behavior of the system. For one value, the behavior might be periodic. For another value only slightly different from the first, the behavior might be completely aperiodic.

2.2 Chaos

In the context of nonlinear dynamical systems, chaos is a word used to describe the time behavior of a system that is aperiodic, and is *apparently* random or "noisy". But, underlying this chaotic randomness is an order that can be determined by the time evolution equations that describe the system. Even when it may sound paradoxical, such an apparently random system is in fact deterministic. Chaos is identified by hypersensitive dependence on initial conditions. A minute change in initial conditions can lead to drastically differing dynamical evolution of the system.

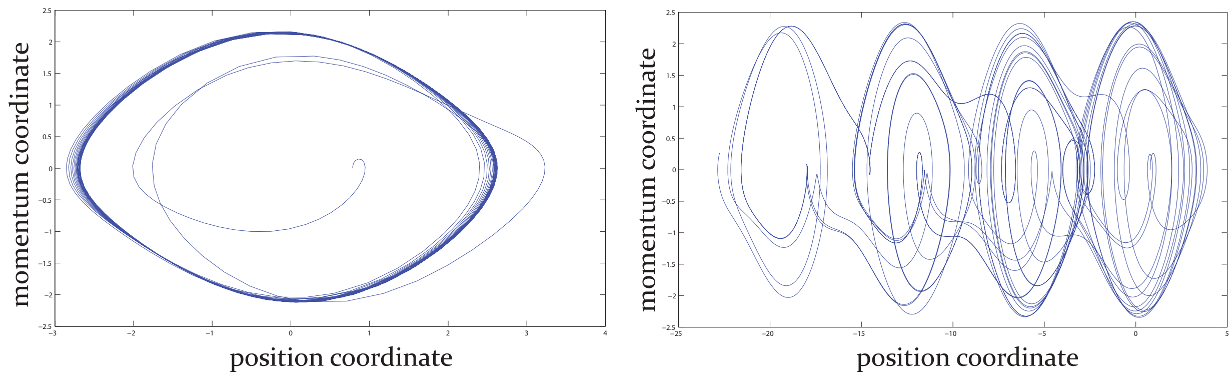
Generally, chaos starts with a so called *period-doubling bifurcation*: system switches to a new behavior with twice the period of the original system at a particular value of a certain parameter. As the value of that parameter is further increased, successive bifurcations occur and the behavior of system takes a time period that is four times, then eight times and so on, finally ending up in chaotic behavior.

The math ingredient: A dynamical system is expressed by its differential equations. Consider what happens to the solution of the system equations when a bifurcation occurs?

2.3 Phase portraits

The notion of state space (or phase space) is a very rich topic. It helps in the quantitative inspection of dynamical systems.The phase portrait is a trajectory of the system, shown in time, with two conjugate variables plotted against each other. For example, for a periodic system exhibiting simple harmonic motion (e.g., a pendulum), the phase portrait will be a closed loop for a particular set of initial conditions. The conjugate variables for the pendulum are the angle θ and its derivative ,$\dot{\theta}$, which is proportional to the angular momentum. For a chaotic system, there will be many distinct loops in a phase portrait, showing that the system is aperiodic and does not approach a stable trajectory.

2.4 Example of a simple pendulum



(a) Periodic Behavior (b) Chaotic Behavior

Figure 1: Possible phase portraits of periodic and chaotic behavior. Refer to main text for labeling of axis.

Consider a simple pendulum as shown in Figure 2 having a small amplitude of oscillation (so that we can assume $\sin\theta \approx \theta$). Ignoring friction, it may be represented using Newton's second law by a normalized second order differential equation of the form:

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\theta = 0 \tag{2}$$

where θ represents the angular position of the pendulum. The solution of this equation is:

$$\theta = \theta_o \sin(\omega t + \phi) \tag{3}$$

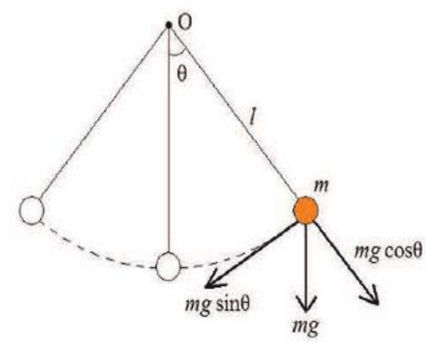


Figure 2: The simple pendulum.

where θ_o is the maximum angular displacement. The first derivative of θ is:

$$\dot{\theta} = \theta_o \omega \cos(\omega t + \phi) \tag{4}$$

Now, from (3) and (4), writing an equation in terms of θ and $\dot{\theta}$ will give the parametric equation:

$$\frac{\theta^2}{\theta_o^2} + \frac{(\dot{\theta})^2}{(\omega\theta_o)^2} = 1 \tag{5}$$

which is evidentally the equation of an ellipse with θ on the horizontal and $\dot{\theta}$ on the vertical axis and represents a periodic trajectory in the phase space. In this context, θ and $\dot{\theta}$ represent the canonical coordinates.

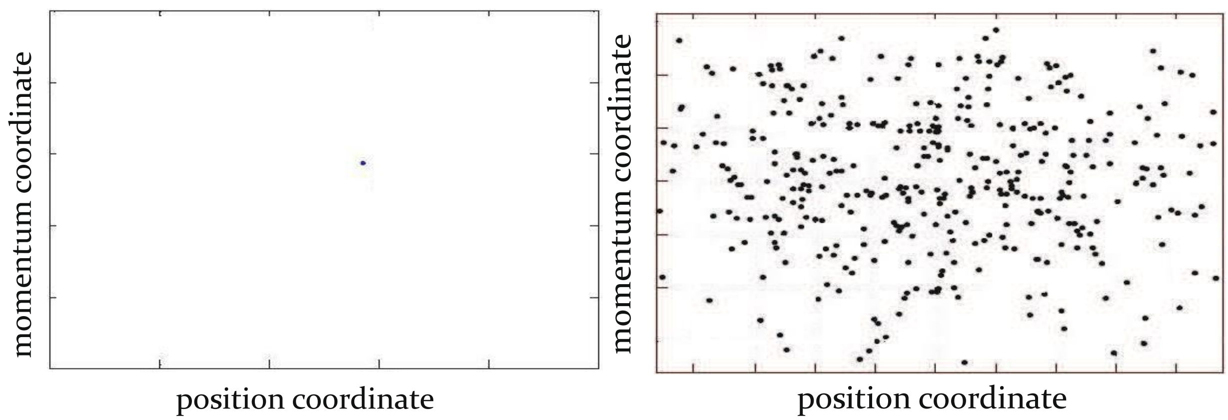
Implicate: Write down the equation of energy of a pendulum in terms of position and momentum variables indicated in the formalism above. What is the total energy in the system?

Figure out: What does a closed loop in phase space signify? What can we say about the energy contained in a system?

A step ahead: Draw the circuit diagram of an RLC circuit. Write down the differential equation of the system and identify the canonical coordinates.

2.5 Poincare sections

Another very useful way of analyzing the behavior of a nonlinear dynamical system is a Poincare Section or Poincare Map. The basic motivation behind making such a map is to reduce an n-dimensional system to an $(n-1)$-dimensional system, making the analysis easier.



(a) Periodic Behavior (Period-1)

(b) Chaotic Behavior

Figure 3: Poincare sections for periodic and chaotic behavior.

Constructing a Poincare map is simple: sample the phase portrait of the system stroboscopically [7]. For periodic behavior, the Poincare map will be a single point or a well-identified distinct groups of points.For chaotic or aperiodic behavior, there will be many irregularly distributed points in the map.

2.6 Bifurcation diagrams

Yet another method of expressing the behavior of a dynamical system over the entire range of a particular parameter is the bifurcation diagram. It shows a correspondence between the parameter values and the resulting response of the system. Every bifurcation indicates a successive period doubling and the response branches off into two. For example, in figure (4), as the control parameter λ is varied over a certain range, the response x_n takes different number of values: two values at the first bifurcation, four values at the second bifurcation, eight values at the third bifurcation and so on. The fuzzy bands indicate chaotic behavior. Also, one can observe the periodic bands within the chaotic ones, showing that chaos can suddenly vanish and give rise to certain higher order periods. This is mainly because of the fact that differential equations defining the system may abruptly switch from chaos to a definite set of solutions for a certain value of the control parameter.

2.7 Universality of chaos and the Feigenbaum Constant

When we look at a bifurcation diagram, such as the one shown in figure (4), we can see the distances between successive bifurcations getting smaller and smaller in a geometric way (along the horizontal axis). This is what Feigenbaum noticed: *the ratio of differences of parameter values at which successive bifurcations occur is the same for all the splittings* [2]. Mathematically speaking:

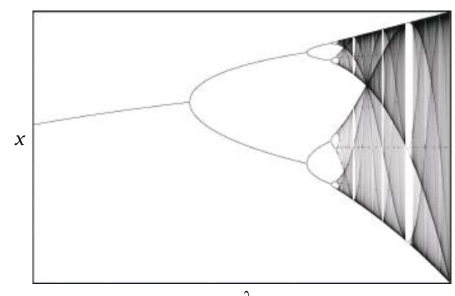


Figure 4: Bifurcation diagram showing the response χ as a function of the control parameter λ. (source: wikipedia.org)

$$\delta_n = \frac{\lambda_n - \lambda_{n-1}}{\lambda_{n+1} - \lambda_n} \qquad (6)$$

where λ_n is the parameter value at which the n'th bifurcation occurs. Moreover, this ratio converges to a particular value called the Feigenbaum constant as n approaches infinity:

$$\delta \equiv \lim_{n\to\infty} \delta_n = 4.669201\ldots \qquad (7)$$

2.7.1 Attractors and fractals

An important manifestation of the fact that chaos is deterministic are *attractors*: a set of points (or trajectories) to which all other trajectories—that start from the initial conditions lying within a region called the *basin of attraction*—approach, as time goes to infinity. Looking at the accompanying figure, we can observe how trajectories remain within a certain region of state-space. This *confinement* of trajectories within a certain region for a particular set of initial conditions is what points toward the determinism in the chaotic behavior.

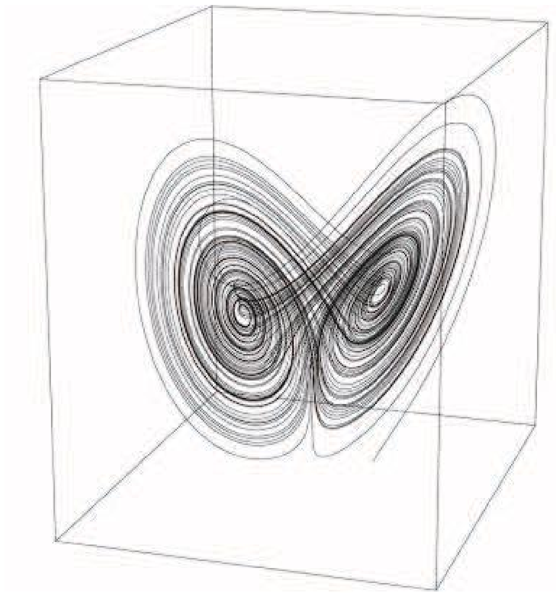


Figure 5: The Lorenz attractor: state-space trajectories are confined.

Attractors, in addition to their aesthetic appeal and tendency to provide us with information about the active degrees of freedom in a system, also determine the dynamical properties of the system's long-term behavior.

(a) Mandelbrot Fractal **(b) Magnified x6** **(c) Magnified x100**

Figure 6: Mandelbrot fractal: regions indicated in boxes are magnified. A resemblance with unscaled image can be noticed even when magnified 100 times.

The discussion on attractors cannot go without mentioning one of the most aesthetically rich notions in mathematics, namely fractals, that actually link attractors with the universality of chaos. Fractals are self similar and self replicating geometrical structures (figure (8)) that occur in the state space as attractors with a *noninteger dimension* and are sometimes called *strange attractors*. Noninteger dimension refers to the idea that, in general, these geometrical figures do not have a point, axis or plane of symmetry, and yet they are self-similar within themselves: they look the same at any degree of magnification.

3 The Experiment

The subject of this experiment is a simple RL-Diode circuit. Although simple it may seem, yet it exhibits interesting behavior including bifurcations and chaos. A series arrangement will be used as shown in the accompanying figure.

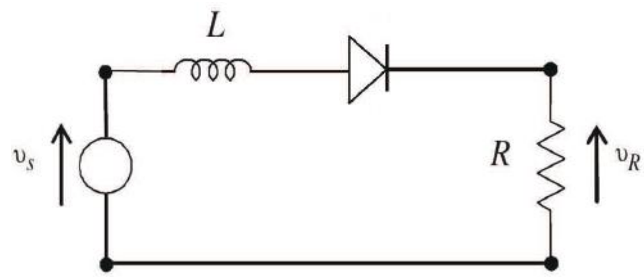
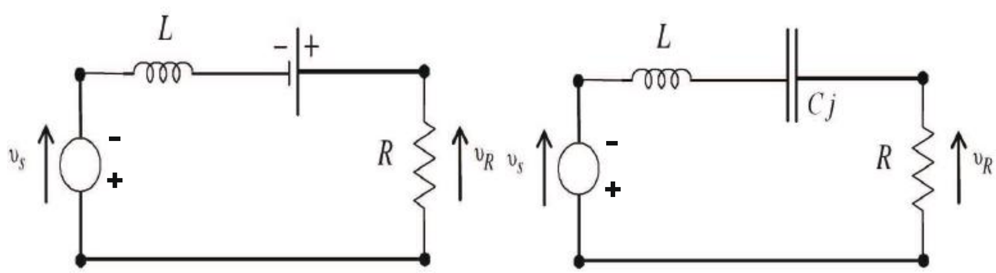


Figure 7: The Experimental RL-Diode circuit.

3.1 The Circuit

The circuit (figure (7)) will alternately behave in two different modes when subject to an AC voltage source: first when the diode is forward biased, the other when it is reverse biased.



(a) Diode forward bias.

(b) Diode reverse bias.

Figure 8: Equivalent circuits for forward and reverse bias cycle.

3.2 The Mathematical Model

During the conducting cycle, the circuit reduces to what is shown in figure (8a), with the diode acting as a fixed voltage drop, i.e, a battery. The Kirchoff's voltage law applied to this circuit gives:

$$L\frac{dI}{dt} + RI = V_o \sin\omega t + V_f \tag{8}$$

where V_o is the peak amplitude of the AC input voltage and V_f is the diode's forward voltage drop, which is generally about 0.5-1.0 V. The solution of this equation, i.e., the current in the conducting cycle is easily found [4]:

$$I(t) = \left(\frac{V_o}{Z_a}\right)\cos(\omega t - \theta) + \frac{V_f}{R} + Ae^{-Rt/L} \tag{9}$$

In the above equation $\theta = \tan^{-1}(-\omega L/R)$ represents a phase delay; A is a constant of integration to be calculated using the initial conditions and $Z_a = \sqrt{R^2 + \omega^2 L^2}$ is the forward bias impedance of the circuit .

In the non-conducting cycle, the diode behaves as a capacitor having a capacitance equal to its junction capacitance (C_j). The equivalent circuit can be represented as a driven RLC circuit (figure (8b)). The loop equation becomes a second order differential equation:

$$L\frac{d^2 I}{dt^2} + R\frac{dI}{dt} + \left(\frac{1}{C_j}\right) I = V_o \omega \cos\omega t \tag{10}$$

Derive: Derive the above equation from Kirchoff's voltage law.

Equation (10) can be solved using traditional techniques.

Derive: Derive the solutions of Equations (8) and (10). The solution for Equation (8) is given in (9) while the final solution of equation (10) is given below:

$$I(t) = \left(\frac{V_o}{Z_b}\right)\cos(\omega t - \theta_b) + Be^{-Rt/2L}\cos(\omega_b t - \phi) \tag{11}$$

The constants B and ϕ are constants of integration and can be found using the initial conditions of the cycle. Moreover, $\theta_b = \tan^{-1}(\omega R/L(\omega_o^2 - \omega^2))$, $\omega_o^2 = (1/LC_j)$, $\omega_b^2 = \omega_o^2 - (R/2L)^2$ and $Z_b = \left(\frac{L}{\omega}\right)\sqrt{(\omega_o^2 - \omega^2)^2 + \left(\frac{R\omega}{L}\right)^2}$

3.3 The Physical Model

3.3.1 The diode recovery-time

Prior to looking into the practical behavior of the circuit and how it becomes chaotic, we need to understand the meanings and significance of an important parameter: the diode's recovery time. The recovery time of a diode is the time a diode would take to completely stop the flow of forward current through itself as it moves into the non-conducting cycle. It depends on the amount of maximum forward current that has just flown through the diode. The greater the peak forward current, the longer the diode recovery time. Quantitatively speaking the recovery time is given by [4]:

$$\tau_r = \tau_m[1 - \exp(-|I_m|/I_c)] \tag{12}$$

where $|I_m|$ is the magnitude of the recent most maximum forward current, and τ_m and I_c are fabrication parameters for the specific diode.

Bring to Light: What is the physical explanation of a diode's junction capacitance? What relationship does it have with the recovery time?

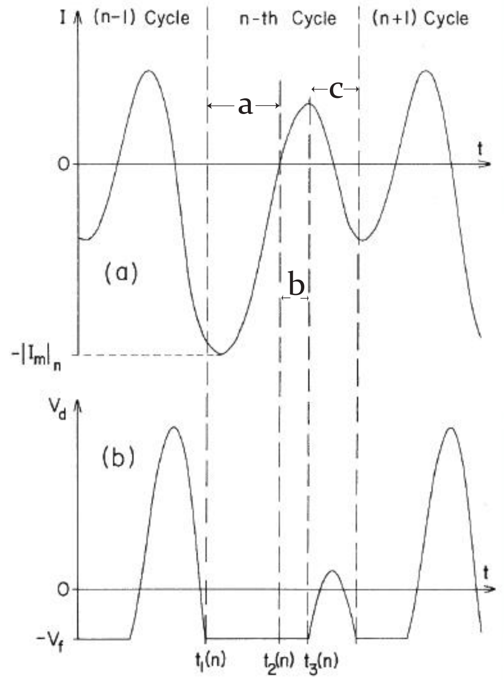


Figure 9: Circuit current,I, and diode voltage, V_d, (period-2) [4]. The diode conducts when $V_d = -V_f$ behaving as in the circuit in figure (10a). Otherwise it behaves as a capacitor as shown in figure (10b).

3.3.2 The period-doubling route to chaos

A physical description of how the RLD circuit leads to period doubling is described in detail in [4]. Here we reciprocate the most important points.

A certain amount of reverse current will flow through the diode in every reverse bias cycle due to the finite recovery time of the diode. If the peak current $|I_m|$ is large in the conducting cycle (figure (9), interval 'a'), the diode will switch off with a certain delay (figure (9), interval 'b') due to the finite recovery time and so will allow a current to flow even in the reverse-bias cycle (shown in the interval 'b'). This reverse current, in turn, will prevent the diode from instantly switching on in the forward bias cycle; it will turn *on* with a delay (figure (9), interval 'c'). This will keep the forward peak current smaller than in the previous forward bias cycle, hence giving birth to two distinct peaks of the forward current. Notice that it took *two* cycles of the driving signal in this process. This is what we identify as a period-doubling bifurcation.

When the peak value of the drive voltage is increased, bifurcation to period-4 may occur followed, possibly, by higher bifurcations and eventually chaos.

Self-Assessment: Briefly explain figure (9) according to the labels on the time axis, describing what happens at every marked instant.

3.4 The Procedure

3.4.1 The Setup

The list of equipment used in this experiment is listed here.

1. Oscilloscope (Agilent DSO-X 2002A)
2. Function Generator (BK Precision 4086)
3. Data Acquisition Setup (National Insturments DAQ card)
4. Spectrum Analyser (Agilent N9320B)
5. RL-Diode circuit components - $100\ \Omega$ Resistor, ≈ 15.054 mH Inductor, 1N4007 Diode

Chaos and bifurcations are usually observed by changing only one parameter (and keeping all others constant) and observing the response of the system. In our case the system is an RLD-circuit in which current and voltage will be made to oscillate using an AC signal. The signal is controlled through two parameters: frequency and amplitude. So we have the option of keeping one of them constant and changing the other one. In our case frequency will be kept constant while amplitude is varied.

It is now time to start our experimental expedition.

Before you begin: Keep the operating manuals of the oscilloscope and signal generator handy. You will have to frequently consult these.

3.4.2 Time-series of voltage across resistor

Connect the circuit in series to the signal generator. Using a BNC-to-Crocodile clips cable, feed V$_R$ into Channel 1 of the Agilent oscilloscope.

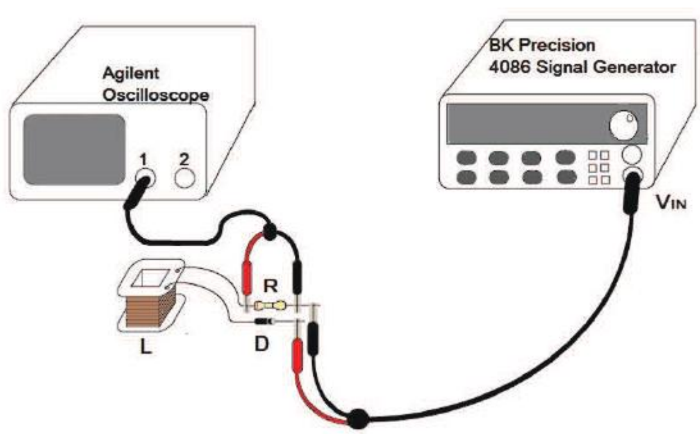


Figure 10: Schematic for the circuit for measuring V$_R$ across the resistor.

Fix the frequency of V_{in} using the BK4086 signal generator to 50 kHz. (**NOTE**: The frequency will be kept fixed for all future segments of the experiment.)

Now change the amplitude of V_{in}in steps of 0.01 V and observe and record the changes in the V$_R$ waveform on the oscilloscope. Keep adjusting your oscilloscope scales accordingly to view the full waveform. You expect to see a period-1 ($\frac{1}{50}$ms) waveform. which transitions to period-2, period-4 and so on (See sample results posted online).

When bifurcations occur you will also need to alter the trigger level (using the trigger knob) of the oscilloscope so that bifurcation peaks are clearly visible on the waveforms. A more stable and finer waveform can be formed by increasing the number of samples (of the input signal at Channel 1) being averaged out by the oscilloscope.The number of averages can be increased by pressing **Acquire** on the oscilloscope and increasing the averaged number to 128.

Questions

1. When does the first bifurcation occur ($2T$)?
2. When the does the second bifurcation occur ($4T$)?
3. Are you able to see the third bifurcation ($8T$)?
   Sketch the waveform plots of V$_R$ for $1T$, $2T$, $4T$ and $8T$ bifurcations and describe them in your notebooks.

TIPS:

1. The voltage values (of V_{in}) at the first point of *onset* of the bifurcation peaks should be set as the voltage values where the bifurcations take place.
2. Do not decrease the amplitude of V_{in} while you are increasing the amplitude to observe bifurcations because voltage values at which bifurcations occur differ for cases where you increase or decrease the amplitude due to hysteresis in the system. Hysteresis is an important feature of nonlinear dynamical systems.

3.4.3 Time-series of voltage across diode.

Connect the circuit in the same way before except that in this case you will be feeding V_D to channel 1 of the oscilloscope. You will also have to reverse terminal connections inside the circuit as shown in the schematic below. Precisely following the color scheme below will help!

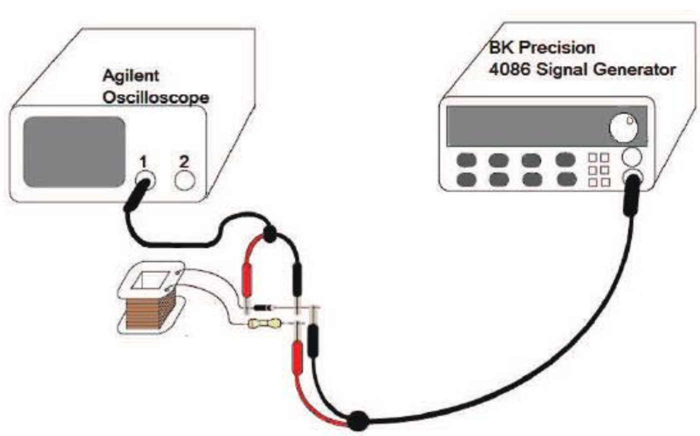


Figure 11: Schematic for measuring V_D across resistor.

Repeat the whole process as you did for waveform time-series of V_R. And note down the voltage values at which bifurcations occur.

Question How do your voltage values compare for bifurcations compare for V_R and V_D?

A diode has nearly a finite potential drop, V_R, when its forward biased (in the case of 1N4007 it is 1 V). In the reverse bias mode the diode the does not conduct (although it does become conducting after the point of blocking voltage has been passed). Now we are going to introduce a DC offset in our V_{in} and study the effects of it on V_D waveform visible on the oscilloscope.

Select a voltage value now where you can clearly see a stable second bifurcation ($2T$) on the waveform. Now introduce a DC offset into your V_{in}. First introduce an offset of $+1$ V and then an offset of -1 V into your signal.

To introduce an offset using the signal generator press **Shift** and then press **Offset/Gate** and using the knob select your amplitude for the offset.

Question What happens to the peaks in the case of $+1$ V offset? In the case of -1 V? Record and explain your observations.

3.4.4 Observing period bifurcations on the spectrum analyzer and determining the Feigenbaum constant.

As it is very hard to determine and clearly observe the point at which the third bifurcation occurs we will now use a spectrum analyzer. It is much easier to identify the third bifurcation by seeing the Fourier spectrum of the signal. Therefore the spectrum analyzer will help us in identifying the different frequencies present in the signal by creating the the spectral density graph and help identify the voltage values for the respective bifurcations.

Calculate Use Equation (6) and voltage values for $2T$ and $4T$ from previous sections to calculate and predict the voltage at which the third bifurcation should occur ($8T$).

The amplitude will again be varied manually in steps of 0.01 V using the signal generator but this time the signal V_R (across the resistor) will be fed into the spectrum analyzer.

To set up the spectrum analyzer follow the following steps:

1. After you have turned the analyzer on press **Preset/System**.
2. Now press **Preset**. This will take the analyzer to its factory settings.
3. To calibrate the analyzer first connect a BNC cable between **CAL OUT** and **RF IN**.
4. Then press the buttons in the following sequence **Preset/System→ Alignment→Align→ All**.
   The system will take a minute to align itself.

Here are a few tips for using the spectrum analyzer.

1. Now connect the BNC which gives the voltage output across the resistor (V_R) and connect it to **RF IN**. Connect all the circuit components in the same way as shown in the schematic below.
2. To change the width of the spectral window press **Frequency**. Then press **Stop Frequency** and **Start Frequency** to set frequency limits of the window.
3. To make the peaks clearer and more distinguishable press **Amplitude**. Then press **Scale Type** and press it again to select **Lin**. Now press **Ref Level** and turn the knob to adjust the reference level.
4. To change the band width resolution press **BW/Avg** and all the options will be in front of you.
5. Set the number of **Average** samples to be 30 and set the **Average type** to power.
6. You can check the amplitude and the frequency corresponding to each peak by **Peak Search** and then switching between the peaks. Our if you want to go to a certain point on the spectrum press **Marker** and then rotate the knob to move the marker around the spectrum.

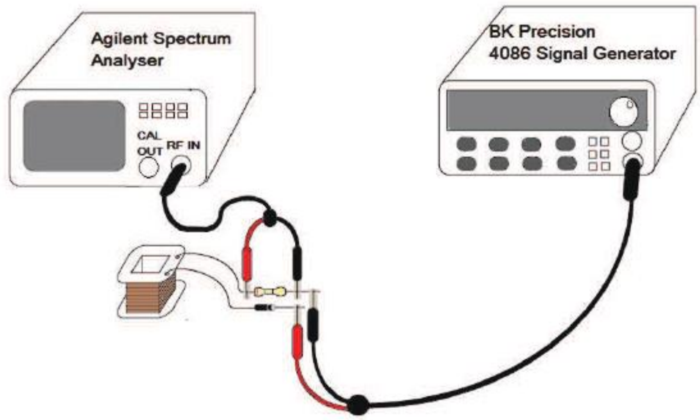


Figure 12: Connecting the circuit to the spectrum analyzer.

Explain The peaks which arise when bifurcations occur always arise to the left side of the main signal peak (50 kHz). Where do the peaks at the right of the main peak come from?

Observe Using the spectrum analyzer you have to note down the value of voltages at which the three bifurcations occur. Note down the frequency and the amplitude of each of the spectral peaks for each of the bifurcations.

HINTS & TIPS

For period one ($1T$) you will see one peak in the spectrum. For period two ($2T$) you will see two peaks one at 50 kHz and one at $\frac{50+0}{2} = 25$ kHz. For period four ($4T$) you will see four peaks and their respective frequencies will be 50 , $\frac{50+25}{2} = 37.5$,25 and $\frac{25+0}{2} = 12.5$ (all in kHz).

Calculate What will be the frequency values of each of the 8 peaks you expect to see in period eight?

For each new wave, you might have to change the reference level, the spectral window, and the resolution of the bandwidth of the spectrum analyzer to be able to see the peaks clearly. For example while observing the point of onset for 25 kHz (period-2) peak, you have to set the reference level to $112.1\ \mu$V, spectral window to 24 - 26 kHz and the bandwidth resolution to 30 Hz. Whereas for period-8 peaks, you will have to decrease the reference level and increase the bandwidth resolution.

Question What is your value for the Feigenbaum constant? Do your voltage values for the bifurcations taken using the oscilloscope and the spectrum analyzer match?

3.4.5 Plotting phase portraits.

Connect the circuit as shown in the schematic diagram. (Figure 13):

1. Turn on the oscilloscope and the signal generator.

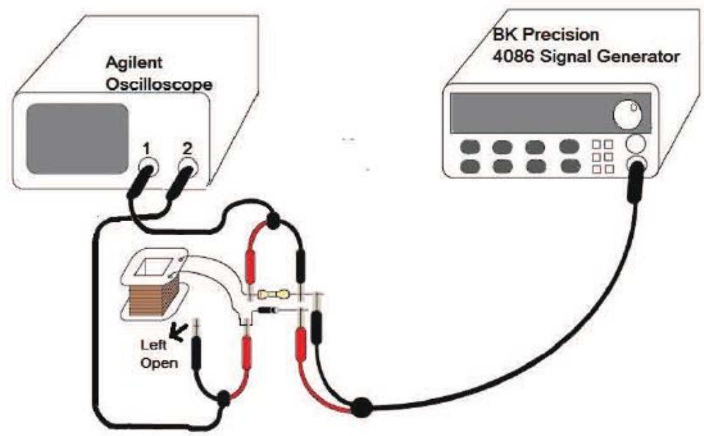


Figure 13: Schematic for the circuit for observing phase portraits on the oscilloscope.

2. Bring the oscilloscope into XY mode. On the oscilloscope you will now be observing the phase portrait of this RLD-circuit. Channel 2 shows a signal proportional to the current I, and channel 1 is connected to a signal proportional to $\frac{dI}{dt}$, which are the variables for this system.
3. Start increasing the frequency from 0 V$_{p-p}$ and keep increasing it to 20 V$_{p-p}$ in steps 0.01 V.
4. Observe the changes in the phase portrait. Keep adjusting your oscilloscope scales accordingly to view the full phase portrait clearly.

Question

1. When does the first bifurcation occur ($2T$)?
2. When the does the second bifurcation occur ($4T$)?
3. Can you see period eight? Sketch and record all your phase portraits on your notebook.

3.5 Data acquisition and processing

In this final step, we will make a phase portraits, Poincare maps, and bifurcation diagram. For this purpose, we will automatically sweep the input voltage (V_{in}) in a linear fashion. Using the NI DAQ card, a up-ramp voltage pulse will be generated, which will modulate the sine wave produced by the function generator ($BK4086$).

First perform the following steps in order to configure the signal generator (BK 4086) enabling it to perform amplitude modulation :

1. Press **[AM]** to activate AM mode.

2. Press **[frequency]**, then **[50] [kHz]** to set the frequency of the carrier wave. This can also be done by turning the knob.

3. Press **[amplitude]**, then **[2] [0] [V]** to set the amplitude of the carrier signal. This can also be done by turning the knob.

4. Now when you press **[menu] AM LEVEL** (modulation depth) will appear on the screen. To set the **AM LEVEL** press **[1] [0] [0] [N]**.

5. Press **[menu]** again and now **AM FREQ** will appear on the screen and you have to select the frequency/timeperiod of the increasing-ramp wave. As we will be using an external source (DAQ card) for modulation therefore no specific value has to be set for **AM FREQ**.

6. The same will be the case for **AM WAVE**.

7. Lastly press **[menu]** to select the **AM SOURCE** and set it to external by pressing **[2] [N]**.

8. Now to finish off the process press **[menu]** again.

3.5.1 Mastering Simple Modulation

Before plotting the bifurcation diagram, it is recommended for you to perform a simple exercise to understand how amplitude-modulation works.

Connect the RLD-circuit with the oscilloscope as you did for in the case of a phase portrait in the previous section. In this case you will also need to connect the DAQ card output terminals to the BNC input terminal (for external modulation) that goes to the back of BK 4086. You will need a BNC-to-crocodile clips cable. The BNC cable will be attached to the **MOD IN (**3 **V**$= 100$ **%)** terminal on BK 4086. The black clip of the cable will be attached to the orange (-1.5 V) DAQ terminal wire and the red clip will be attached to the green ($+1.5$ V) DAQ terminal wire.

Turn on your computer and open the LabVIEW file on the desktop under the name '`simple_modulation`'. The modulating sine wave that is being generated by the LabVIEW program and delivered to the signal generator via DAQ has a frequency of 500 Hz and a voltage of 3 V_{pp}.

Simple Modulation Exercise

1. Double check if the modulating wave is a sine wave with frequency and amplitude as mentioned above.

2. Run the LabVIEW file and record the maximum and minimum voltage values for the modulated wave on the oscilloscope.

3. Do you know what is a modulating single, carrier signal, and a modulated signal, and where are they being produced in our system?

4. While performing the settings for the signal generator, you set the AM LEVEL to 100%. Now change it to 75%, then to 50%, and finally to 25% and record the maximum and minimum voltage values.

Filename	Functions
RLD_DAQ.vi	It generates a amplitude-modulation ramp that is fed into the circuit. Once the signal starts appearing clearly on the oscilloscope, which takes around 40 seconds, the LabVIEW file starts recording two sets of data: one across the resistor (V_R) and the other across the resistor + inductor ($V_{\mathrm{R+L}}$) and stores them in separate files. Using this data and a peak detection algorithm, it then creates time-series plots, phase portraits and Poincare maps. Figure 14 shows its front panel.
Data_Acquisition.vi	By storing voltage across resistor, it create a file of V_R with corresponding V_{in} values. This is extremely important as the bifurcation diagram is essentially a plot between these two parameters. It might seem that these two sets of data should automatically be stored side by side. However, V_{in} is the input signal whereas V_R is the output signal so a mechanism is needed to store the two together. There is also Savitzky Golay Filter to fit the data onto a polynomial of choice and minimize noise. There is a button in the front panel to control the filter and the degree of polynomial.
Peak_Detection.vi	It reads the output file from Data_Acquisition.vi and goes through all its data points to search for turning points. Finally, it note downs the value of V_R at a turning point and its respective V_{in} in a new notepad file.

Table 1: LabVIEW files used for creating the bifurcation diagram.

After understanding the modulation mechanism that sends a ramp signal of increasing V_{in} into our circuit, we will move towards data acquisition and processing. For this purpose, we have created three separate LabVIEW files whose functions are explained in Table 1.

3.5.2 Making Phase Portraits and Poincare Maps

Open the **RLD_DAQ.vi** file. The interface would look similar to 14.

1. Set the modulation depth on the signal generator back to 100%.
2. Set the **width** to **40** and set the **Peaks/Valleys** to **Valleys**.

When you will run the VI files, it will take around 15 s for the oscilloscope to start feeding live data to the computer. When the VI has started running you will have to adjust the scales on the oscilloscope ASAP.

IMPORTANT: These adjustments would have to be done using the oscilloscope knobs before 47 s mark on the VI clock. At the 47 s mark our modulated pulse will be initiated.

1. Set the Channel 1 amplitude scale to 500 mV
2. Set the Channel 2 amplitude scale to 50 V.

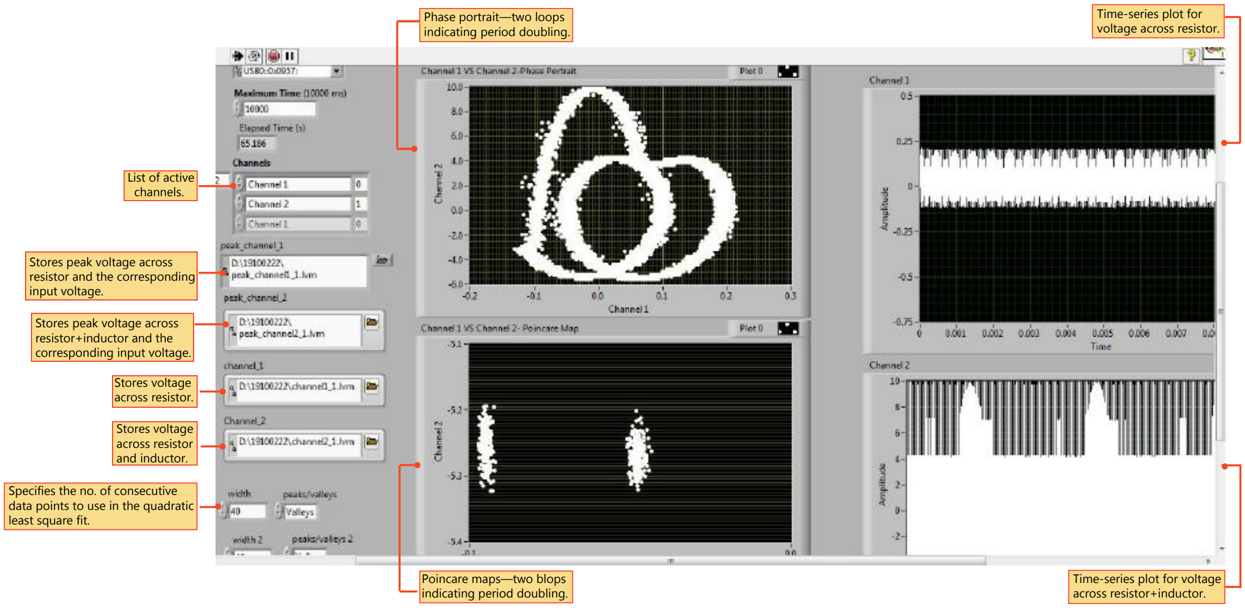


Figure 14: `RLD_DAQ.vi`'s interface on the computer.

3. Set the horizontal time scale to 5 MSa/s.

Now run `RLD_DAQ.vi`. The file will generate a modulation ramp (50 kHz, 20 V_{pp}) that is fed into the circuit. The ramp pulse will have a period of 20 s and it will change the amplitude of the sine wave from 0 to 20 V_{pp}. Once the signal starts appearing clearly on the oscilloscope, which takes around 40 seconds, the LabVIEW file will start recording two sets of data: one across the resistor and other across the resistor + inductor and store them in separate files. Using this data and the peak detection algorithm, it will also display phase portraits and Poincare maps, which are the most convenient ways of observing chaos.

3.5.3 Drawing Bifurcation Diagrams

In order to plot the bifurcation diagram, we need to identify the peak (or valley) values from a time series of, say, the voltage across the resistor V_R. For period-1, all the peaks have the same amplitude, for period-2 there are peaks of two different amplitudes, for period-4, there are peaks of 4 different amplitudes, and so on. So a parameter (in this case the amplitude, V_{in}) is varied over a period of time and the peaks are sampled. The sampled peak amplitudes of V_R are plotted against the corresponding V_{in}. This gives us the bifurcation graph.

Following are the steps to get a bifurcation diagram for the data,

1. Keeping the set-up as for the last section, run the `Data_Acquisition.vi` file on your desktop. Don't forget to adjust the scales on the oscilloscope for Channel 1, Channel 2 and horizontal time scale as mentioned previously. The VI would run for a total of 70 s.

   As the VI file runs, the sampled data is recorded in the data files specified in the VI interface. The one that should concerns you is the file for 'output file(V_{in}/V_R)'. It has two columns: first for input voltage and second for voltage across the resistor. Moreover, if you look at the interface of the LabVIEW file, you would notice a button for Filter Control. It is recommended to take two sets of data: one with the filter on and one with the filter off.

Question What do you think is the function of Savitzky Golay Filter?

2. Now the next step involves peak detection, for which we will use another LabVIEW file on your desktop, `Peak_Detection.vi`. Here, first you have to load the file for output file(V_{in}/V_R) from `Data_Acquisition.vi`. Set the width to 10, Peak/Valley to Valley, and run the file. Perform this twice for the two sets of data with and without filter. The output from this LabVIEW program gives V_R at which the period changes and the corresponding V_{in}.

3. The last step of the process is to plot the two columns of the output file from peak detection program in matlab. For RLD-circuit, the bifurcation diagram is a plot of V_{IN} against V_R. The bifurcation plot then looks like Fig. 16.

Question Do you notice any difference between the bifurcation plots of the data with and without filter?

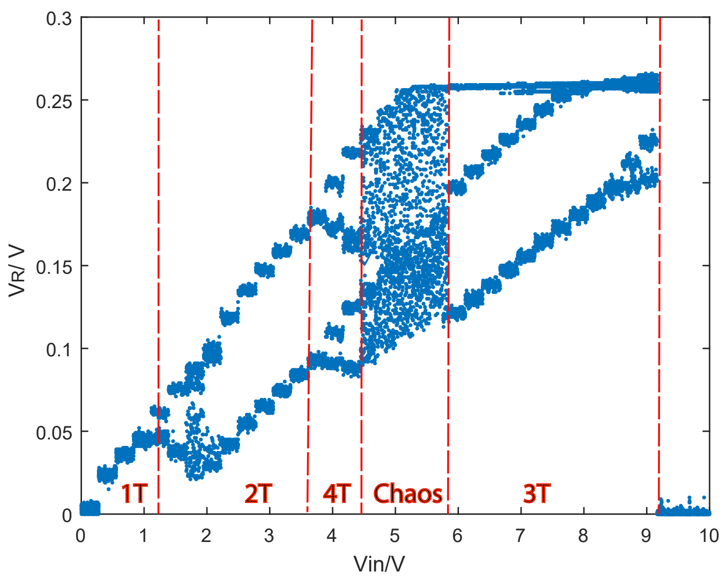


Figure 15: The bifurcation diagram you would obtain after running your VI files.