(b) If the controller is PI, the disturbance error transfer function is

$$T_w(s) = \frac{-Bs}{s^2(\tau s+1)+(k_p s+k_I)A}, \tag{4.58}$$

$$n = 1, \tag{4.59}$$

$$K_{n,w} = \frac{Ak_I}{-B}, \tag{4.60}$$

and therefore the system is **type 1** and the error to a unit ramp disturbance input will be

$$e_{ss} = \frac{-B}{Ak_I}. \tag{4.61}$$

4.3 Control of Dynamic Error: PID Control

We have seen in Section 4.1 basic properties of feedback control, and in Section 4.2 we examined the steady state response of systems to polynomial reference and disturbance input. At the end of Section 4.1 we observed that proportional control changed the time constant of the simple speed-control system. In this section the impact of more sophisticated controls on system characteristic equations is examined in the context of a standard controller structure. The most basic feedback is a constant **P**roportional to error. As we saw in Section 4.2, addition of a term proportional to the **I**ntegral of error has a major influence on the system type and steady-state error to polynomials. The final term in the classical structure term proportional to the **D**erivative of error. Combined, these three terms form the classical **PID** controller, which is widely used in the process and robotics industries.

The PID (proportional-integral-derivative) controller

4.3.1 Proportional Control (P)

When the feedback control signal is linearly proportional to the system error, we call the result **Proportional feedback**. This was the case for the feedback used in the controller of speed in Section 4.1, for which the controller transfer function is

$$\frac{U(s)}{E(s)} = D_c(s) = k_p. \tag{4.62}$$

As we saw in Section 4.1.4, the time constant of the feedback system was reduced by a factor $1+Ak_p$ by proportional control. If the plant is second order, as, for example, is a DC motor with nonnegligible inductance, then the transfer function can be written as

$$G(s) = \frac{A}{s^2+a_1s+a_2}. \tag{4.63}$$

In this case, the characteristic equation with proportional control is

$$1 + k_p G(s) = 0, \tag{4.64}$$

$$s^2 + a_1 s + a_2 + k_p = 0. \tag{4.65}$$

The designer can control the constant term and the natural frequency, but not the damping of this equation. If k_p is made large to get adequate steady-state error, the damping may be much too low for satisfactory transient response.

4.3.2 Proportional plus Integral Control (PI)

Proportional plus Integral control

Adding an integral term to the controller results in the **Proportional plus Integral (PI)** control equation

$$u(t) = k_p e + k_I \int_{t_0}^{t} e(\tau)\, d\tau, \tag{4.66}$$

for which the $D_c(s)$ in Fig. 4.5 becomes

$$\frac{U(s)}{E(s)} = D_c(s) = k_p + \frac{k_I}{s}. \tag{4.67}$$

This feedback has the primary virtue that, in the steady-state, its control output can be a *nonzero* constant value even when the error signal at its input is *zero*. This comes about because the integral term in the control signal is a summation of all past values of $e(t)$. In fact, the integral term will not stop changing until its input is zero, and therefore if the system reaches a stable steady state, the input signal to the integrator will of necessity be zero. This feature means that a constant disturbance w (see Fig. 4.4) can be canceled by the integrator's output even while the system error is zero.

If PI control is used in the speed example, the transform equation for the controller is

$$U = k_p(\Omega_{\text{ref}} - \Omega_m) + k_I \frac{\Omega_{\text{ref}} - \Omega_m}{s}, \tag{4.68}$$

and the system transform equation with this controller is

$$(\tau s + 1)\Omega_m = A(k_p + \frac{k_I}{s})(\Omega_{\text{ref}} - \Omega_m) + TW. \tag{4.69}$$

If we now multiply by s and collect terms, we obtain

$$(\tau s^2 + (Ak_p + 1)s + Ak_I)\Omega_m = A(k_p s + k_I)\Omega_{\text{ref}} + AsW. \tag{4.70}$$

Because the **PI** controller includes dynamics, use of this controller will change the dynamic response in more complicated ways than the simple speed-up we saw with proportional control. We can understand this by considering the