Assignment 1: Isolated Magnetic Moments and Paramagnetism

1. (a) Calculate the gyromagnetic ratio γ and Larmor frequency ω (inside a magnetic field $B = 1$ T) for an electron, muon, proton and neutron. Their Landé g-factors are, respectively -2.002, -2.002, 5.58 and -3.83. Quote their masses in your calculations.

   (b) **Atomic physics background question:** Derive an expression for the Landé g-factor for an ion with orbital and spin quantum numbers L and S respectively. (Refer to Appendix A.9 from Blundell.)

2. (a) Find the magnetization as a function of magnetic field and temperature for a system of spin j, and concentration n.

   (b) Find the susceptibility. Also find the susceptibility in the high temperature limit. Does the Curie law hold?

   (c) Plot the susceptibility as a function of B/T for various values of j including $j = 1/2, 1, 3/2, 10, \infty$.

3. The Landau-Lifshitz-Gilbert equation is given by

$$\frac{d\,\boldsymbol{\mu}}{dt} = \gamma(\,\boldsymbol{\mu} \times \boldsymbol{B}) + \frac{\alpha}{\mu}(\,\boldsymbol{\mu} \times \frac{d\,\boldsymbol{\mu}}{dt}).$$

   Here α is a constant that characterizes the damping of the magnetic moment.

   (a) Show that the above equation can be written as,

$$\frac{d\,\boldsymbol{\mu}}{dt}(1+\alpha^2) = \gamma(\,\boldsymbol{\mu} \times \boldsymbol{B}) + \frac{\alpha\gamma}{\mu}\big(\,\boldsymbol{\mu} \times (\,\boldsymbol{\mu} \times \boldsymbol{B})\big). \tag{1}$$

   (b) Write a computer program that simulates the damping of the moment for various choices of α and initial conditions on $\boldsymbol{\mu}$. Choose dimensionless units, $B, \gamma = 1$. For this simulation, a set of coupled differential equations must be solved. An example program (in `Matlab`) that solves the phenomenological Bloch equations has been uploaded on the website.