A Doubly Suspended Pendulum

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Pendulums have been around for thousands of years. The ancient Chinese used the pendulum principle to predict earthquakes. Galileo Galilei was the first European to work on pendulums and discovered that their regularity could be used for keeping time, leading to the first clock. In 1656, the Dutch inventor and mathematician, Huygens successfully built an accurate clock based on the pendulum. Pendulums are used to understand the relationship between gravitational forces and the mass of objects, the changes in speed and direction of objects, as well as the distance between objects.

Figure 1: Zhang Heng's seismoscope, the first device to detect an earthquake [3].

Various kinds of pendulums have been investigated in modern times, such as the simple pendulum, double pendulum, spring pendulum, Foucault pendulum, Kater's pendulum, and spherical pendulum [1, 2]. Nowadays, the conventional pendulum is widely used in engineering, such as energy harvesting and robot design. The intent of the experiment is to investigate the motion of one particular kind called the bifilar pendulum.

KEYWORDS

Bifilar Pendulum · Rigid body · Moment of inertia · Torque · Simple harmonic motion.

APPROXIMATE PERFORMANCE TIME 4 **Hours**

1 Conceptual Objectives

In this experiment, we will,

1. review the equations that govern the dynamics of a doubly suspended pendulum,
2. learn how to compare theoretical predictions with experimental observations,
3. run through a complete cycle of experiment, data generation, data analysis, uncertainties calculations and presentation of results.

2 Introduction

2.1 The bifilar pendulum

A bifilar pendulum consists of a symmetric object (such as a uniform rod) suspended from two parallel wires called filars. These filars allow the body to rotate freely about a given axis shown by $\hat{z}$ in Figure (2). Suppose the suspended rod has an overall length $D = 2$r and each filar has an untwisted length L. Normally, for vivid visual effects, it is recommended that the filar length L is longer than $2r$, preferably 3 or 4 times longer [4, 5].

If the pendulum is twisted in the horizontal plane, then clearly there can be no acceleration in the $\hat{z}$ direction. There are, therefore, no unbalanced vertical forces. The tension in each filar will therefore be $(\frac{mg}{2})\cos\phi \approx \frac{mg}{2}$. Here ϕ is a small angle in the vertical plane indicating the amount of twist. The corresponding twist in the horizontal plane is measured as θ. Figure ($2b$) shows another view of the twisted geometry that clarifies the definitions of these angles. From the geometry, it is also clear that $L\sin\phi = r\sin\theta$. The length d depicts the displacement in the horizontal plane. Furthermore, we have ($\tan\theta = d/r$) and if the twist is small, then $\theta \approx d/r$.

If we release the twisted pendulum, it will revert to its equilibrium position. Where is the restoring torque coming from? It is coming from none other but the horizontal component of the tension $(mg/2)\sin\phi$ with a moment arm r about the axis of rotation $\hat{z}$. The force is depicted by a thick arrow in Figure (2b).

Hence the overall torque acting on the suspended rod is given by,

$$\tau = 2\times\left(\frac{mg}{2}\right) r\sin\phi = mgr\sin\phi \tag{1}$$

The prefactor of two comes from the fact that the restoring torque on both ends of the rod additively combines to create an oscillation in the same direction.

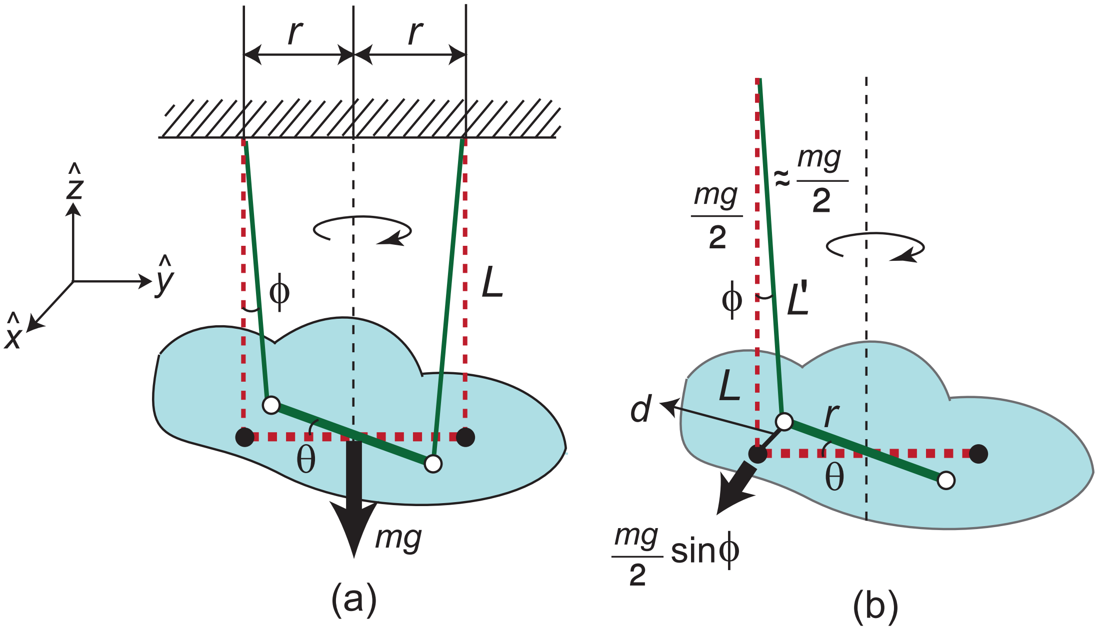

Figure 2: (a) The bifilar pendulum. The angle θ is in the horizontal ($\hat{x}$-$\hat{y}$) plane while ϕ is defined in the vertical plane. The rod with black ends shows the untwisted and the rod with white ends shows the twisted position. (b) Filar suspension geometry.

Newton's second law for angular motion is given by,

$$\tau \quad = \quad I\alpha = I\left(\frac{d^2\theta}{d\,t^2}\right), \tag{2}$$

where I is the moment of inertia and $\alpha = d^2\theta/dt^2$ is the angular acceleration. Notice the analogy of Equation (2) with its linear counterpart $F = ma = m(d^2x/dt^2)$.

Therefore comparing Equations (1) and (2) will immediately yield the angular equation of motion but wait, there is one last step. Equation (1) expresses the motion in terms of ϕ whereas Equation (2) contains θ. It is necessary to interconvert from ϕ to θ. This can be achieved by simple geometric arguments. Observe Figure (2b), and therein the vertical and the horizontal triangles. Notice that from the vertical triangle we have,

$$d \quad = \quad L^{'} \sin\phi \;\approx\; L\sin\phi, \tag{3}$$

where $L^{'}$ is the twisted length and since ϕ is small, $L^{'} \approx L$. From the horizontal triangle,

$$d \quad = \quad r\tan\theta \approx r\sin\theta \approx r\theta, \tag{4}$$

and by comparing Equations (3) and (4), we obtain,

$$\sin\phi \approx \left(\frac{r}{L}\right)\theta. \tag{5}$$

Substituting into Equation (1) and finally comparing with Equation (2) yields,

$$\frac{d^2\theta}{d\,t^2} + \left(\frac{mgr^2}{I\,L}\right)\theta \quad = \quad 0. \tag{6}$$

Compare this expression with the expression for simple harmonic motion, such as of a simple pendulum [6],

$$\frac{d^2\theta}{d\,t^2} + \left(\frac{g}{L}\right)\theta \;=\; 0. \tag{7}$$

Rearranging the above expression yields the time period of a bifilar pendulum,

$$T \;=\; \frac{2\pi}{r}\sqrt{\frac{L\,I}{m\,g}} \tag{8}$$

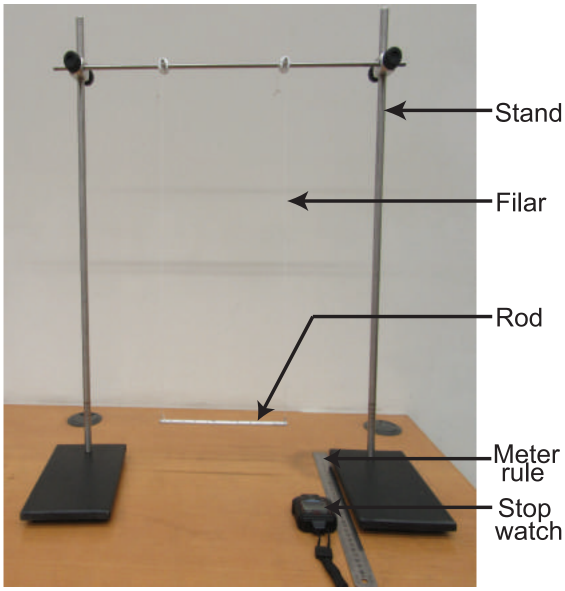

Figure 3: The experimental setup for a bifilar pendulum.

Q 1. How is a bifilar pendulum useful in everyday life and why do we study about it?

3 Experimental Method

The objective of this project is to investigate the moment of inertia of a bifilar pendulum experimentally and then comparing it to the theoretical prediction, posited in Equation (8). The investigation of the bifilar pendulum includes determining the accuracy of the moment of inertia inferred from the experiment and understanding the underlying experimental process. The necessary apparatus is provided to you and photographed in Figure (3). All of this is self explanatory.

Measure the weight and length of the object of interest which is the rod. Disturb the equilibrium position of the rod by a gentle push in a way that the rod twists about the centre of mass. Generate your own data, analyze and quote uncertainties. You should sketch and log the procedure appropriately in your notebooks.

Q 2. Predict the moment of inertia of the rod using its particular geometry. Quote the uncertainty.

Q 3. Measure the time period of the twisting rod using stop watch. What is the uncertainty in T?

Q 4. Determine the moment of inertia of the rod and its uncertainty.

References

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