Polarization Peanuts with Fourier Analysis*

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In many optical experiments, it is extremely important to accurately determine the polarization state of light. The complete polarization state of light can be described in terms of the Stokes parameters, which are related to the polarization ellipse. This experiment is one of the simplest schemes of determining the Stokes parameters.

Essential pre-lab reading: "*Field Guide to Polarization*" by Edward Collett, SPIE Press, 2005; (pp. 7-14).

1 Test Your Understanding

1. Derive the Stokes parameters in terms of tilt and ellipticity angles of the polarization ellipse. Determine the Stokes parameters for elliptical, circular and linear polarization states.

2. Using a linear polarizer and quarter wave plate (QWP), outline a scheme for generating elliptical, circular and linear polarizations at arbitrary angles. Validate it using Jones calculus.

3. Derive the intensity profile for the experimental setup shown in Figure 1. Simplify and write the answer in the form of a Fourier series in the variable α, the analyzer angle. The Fourier series will have the form

$$I(\alpha) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\alpha) + \sum_{n=1}^{\infty} b_n \sin(n\alpha).$$

   What are the Fourier coefficients? Your answer should show that only three of the Fourier coefficients are non-zero. The Fourier coefficients should be in terms of polarizer and QWP angles, β and θ respectively.

4. Based on your scheme in Step 2, plug in values of polarizer and QWP angles in the Fourier coefficients. What set of values of the Fourier coefficients do you get for circular, elliptical and linear polarizations? How do these Fourier coefficients compare with the Stokes parameters (calculated in Step 1)?

 We would like to thank Zahra Tariq and Mah Para Iqbal for helping write this lab manual.

5. For different polarizer and QWP angles, make polar plots with intensity treated as radius shown against the variation of the analyzer angle. Another way of looking at the polar plot between intensity I and angle α is plotting $I \cos \alpha$ versus $I \sin \alpha$.

6. Observe the plots generated in the previous step. They would generally be peanut-shaped figures. Could you correlate the shape of the plot with the input polarization state falling on the analyzer?

2 The Experiment

A HeNe laser beam passes through a polarizer and then a QWP. The beam emerging the QWP passes through an analyzer and a neutral density filter (NDF) and finally falls on the photodiode (PD), as shown in Figure 1. The polarizer and the QWP form the polarization generator while the analyzer and the PD constitute the polarization analyzer.

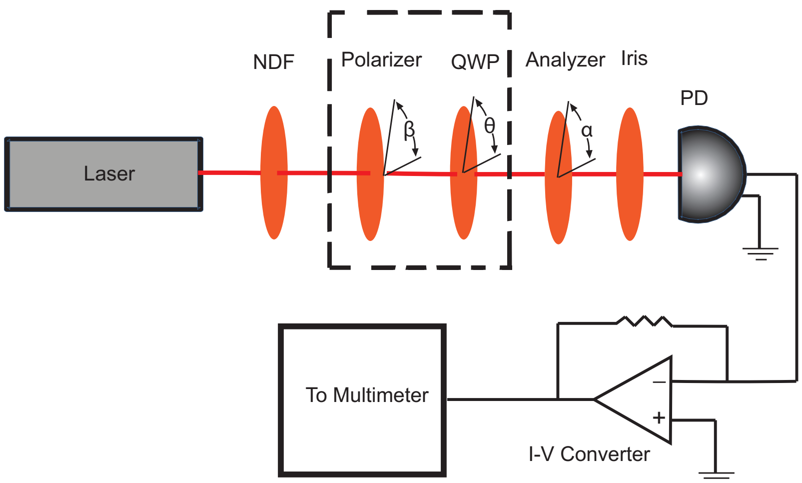

Figure 1: Setup for the polarimetry experiment. The red line shows the perceived light path. The angles mentioned along with the optical elements are referred with respect to the horizontal axis of the lab. The dashed box shows the section of the apparatus that generates light of some polarization and the Stokes parameters. We use the analyzer to measure this polarization.

Orient the polarizer such that it produces horizontal polarization. Hence β is nominally $0°$ in the remainder of this experiment. All angles will then be measured with respect to this polarizer. To make sure that the beam coming out of the polarizer is horizontally polarized, follow this tutorial: `https://goo.gl/zGxphV`. Orient the QWP to an arbitrary angle. Now, rotate the analyzer from $0°$ to $360°$ in steps of $10°$ and record the intensity profile detected by the PD. Perform the experiment thrice by generating circular, elliptical and linear polarization states.

Q 1. Plot a graph between measured intensity and analyzer angle. Does it agree with your calculations based on the Jones calculus?

Q 2. Determine the Fourier coefficients of the intensity profile. How many are they? Can you determine the polarization of light based on these coefficients?

Q 3. Make a polar plot of intensity versus analyzer angle. What is the shape of the plot? Does it corroborate with your intensity calculations?

Using this polar plot, it is possible to determine the shape of the polarization ellipse and hence the polarization state of the beam.

Q 4. Measure the angle τ between the major axis of the polar plot and the horizontal. Measure the ratio a/b between the major and minor axes of the polar plot and use it to determine the angle $\varepsilon = \pm\tan^{-1}(\sqrt{b/a})$. Use τ and ε as tilt and ellipticity angles, respectively, corresponding to the polarization ellipse and determine the Stokes parameters. Do they agree with your previous calculations?

Q 5. Can you determine the handedness of elliptical or circular polarization? Why or why not?