Conical Swing of a Pendulum and Centripetal Forces

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This experiment uses a conical pendulum to familiarize us with dynamic equilibrium in rotational motion. In this investigation, we will identify the free body diagram of a horizontally whirling object and see how it lifts as its speed goes up. The lift apparently defies the downward pull of gravity but we will see how a delicate balance of forces allows the object to swirl at a certain lift angle. As the speed of the object changes, the forces adapt and preserve the delicate balance.

This experiment eventually becomes a really nice demonstration of the interplay of geometry, vectors, forces, and equilibrium. This experiment recreates the mental problem in Paul Nahin's beautiful book [1], *"In Praise of Simple Physics"* and is one of the author's methods to determine the value of g.

In this experiment, the angular speed of a pair of spheres moving in a horizontal plane is controlled by a stepper motor. The angular speed is monitored in real-time and displayed on a computer screen. We then use an image processing software to find the geometric parameters of the system. This information is used to make quantitative assessments. In short, this experiment clears up concepts about rotational motion and also helps in visualizing how a system keeps up with the laws of physics.

KEYWORDS

Dynamic equilibrium · angular velocity · centripetal force · conical pendulum · acceleration due to gravity.

1 Objectives

In this experiment, we will,

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1. learn how to draw a free body diagram of an object,

2. observe the effect of angular velocity on geometric parameters of conical motion,

3. learn how to use an image processing software to find angles,

4. learn resolution of vectors and their use in equilibrium equations, and finally,

5. learn how to calculate uncertainties when we take measurements from a video.

2 Theoretical Introduction

Have you ever tried swinging a bucket filled with water over your shoulder? Why doesn't the water spill out when the bucket is upside down? Now replace the bucket with a stone. Why is there still tension in the string when stone reaches the top position? What is the minimum angular speed you need in these attempts to pass the critical top point? Retry with a shorter string length; do you need the same effort to reproduce the effect? These simple experiments that some of us have tried in our childhood have simple but deep explanations in basic physics.

In this experiment a small plastic ball is attached at one end of a light aluminum rod while the other end is tied to a circular disk. Two such rods and balls are available, see Fig. 1. The disk is mounted on a metallic shaft attached to a stepper motor's coupler. When the stepper motor starts rotating, the disk also rotates and plastic balls lift . As the plastic ball is moving in circular motion it must be experiencing centripetal acceleration a_c which is given by:

$$a_c = \frac{v^2}{r},\tag{1}$$

where v is the linear speed of the mass and r is the radius of the circular motion. The acceleration is directed along the radius of the dashed circle and towards the shaft.

If we take T as time period of rotatory motion then velocity can also be expressed as:

$$v = \frac{2\pi r}{T}.\tag{2}$$

Now replace the expression of v frrom Eq. (2) into Eq. (1) obtaining;

$$a_c = \frac{4\pi^2 r}{T^2}.\tag{3}$$

The required centripetal force to keep the circulating mass in orbit is provided by the horizontal component of the string tension F:

$$F\sin(\alpha) = m\frac{4\pi^2 r}{T^2}.\tag{4}$$

Furthermore, as there is no vertical motion of mass so the vertical component of the string tension is equal to the weight:
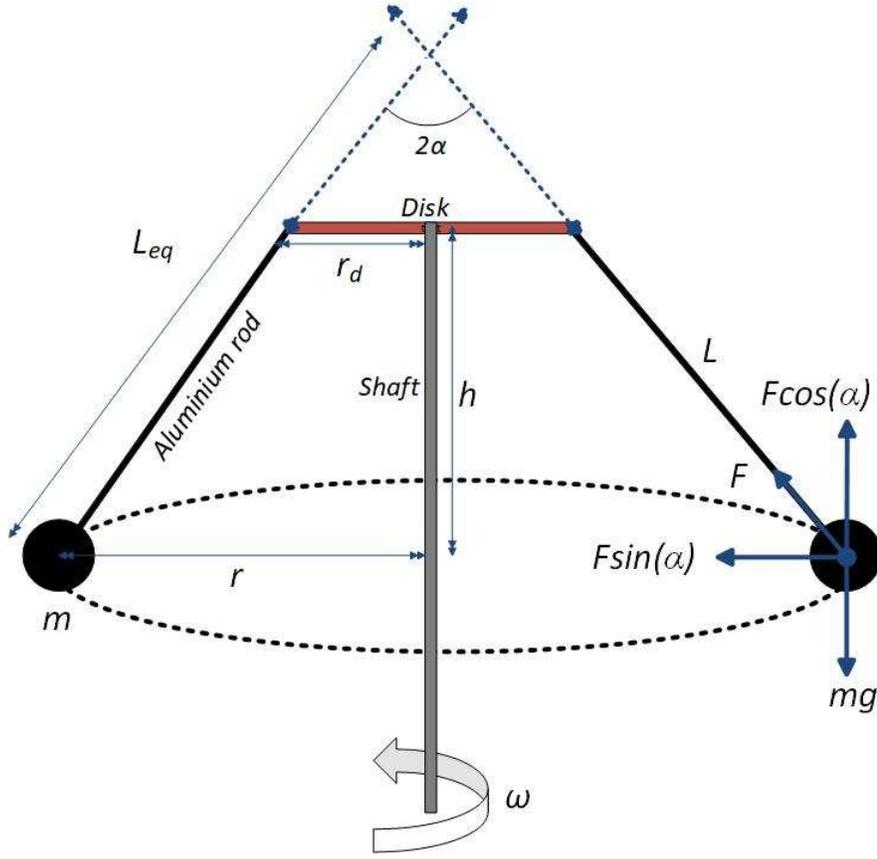
$$F\cos(\alpha) = mg.\tag{5}$$

Figure 1: Schematic diagram of the conical pendulum. The geometry and the variables used in the main text are identified.

This is how the delicate balance is achieved. Note from Eqs .(4) and (5) that if T changes, $F \sin(\alpha)$ and $F \cos(\alpha)$ must readjust. From these equations we can show that:

$$F \sin(\alpha) = \frac{F}{g} \cos(\alpha) \frac{4\pi^2 r}{T^2} \tag{6}$$

and from the geometry, we know that:

$$\sin(\alpha) = \frac{r}{L_{eq}}, \tag{7}$$

$$\text{and} \quad \cos(\alpha) = \frac{h}{L_{eq}}, \tag{8}$$

we eventually arrive at the expression,

$$g = \frac{4\pi^2 h}{T^2}. \tag{9}$$

Note that L_{eq} is the distance of the ball from the apex of rotation and L is the physical length of the rod. We can also identify angular velocity squared term $4\pi^2/T^2$ and replace h with $L_{eq} cos(\alpha)$ leading to the expression:

$$\omega^2 = \frac{g}{L_{eq} cos(\alpha)}. \tag{10}$$

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Q 1. Does this equation impose any minimum threshold to the value of L_{eq}? If yes, then calculate this threshold value for a conical pendulum rotating at 50 revolutions per minute (rpm)? What is the threshold at 10 rpm and 100 rpm?

Q 2. Derive an equation to calculate L_{eq} using the parameters defined in Fig(1). Show that in our setup the threshold value of L_{eq} also depends on the radius of top disk.

3   Apparatus

The apparatus for this experiment includes a stepper motor housing (the black box), a power source and the conical pedulum. The black box houses a stepper motor, Arduino controller and a motor driver to control the angular speed of the conical pendulum. There is a provision of rpm control knob in front of the box. All components inisde box are powerd by PhysVolt15 (DC 15V 2A) power supply (Qosain Scientific).
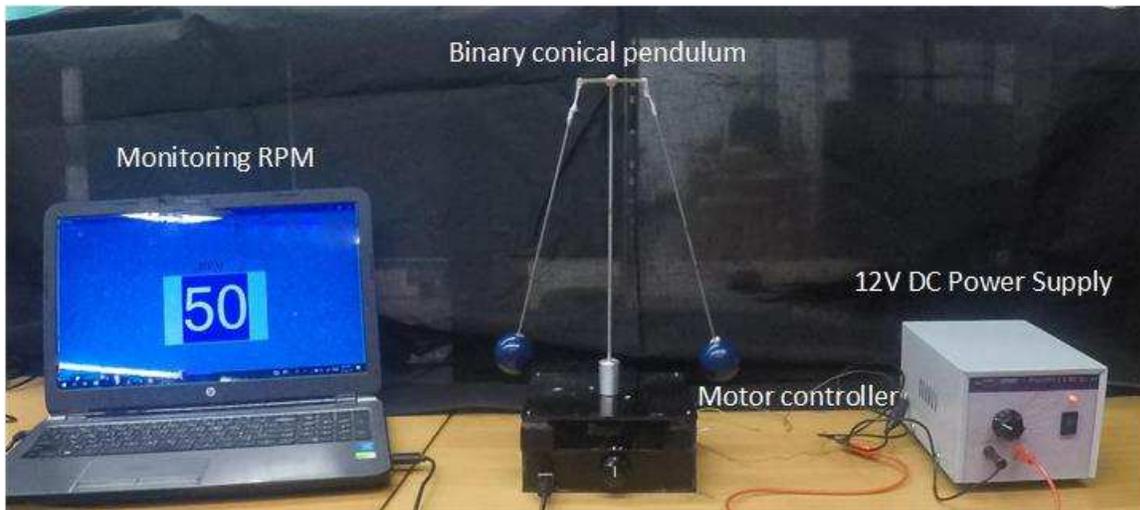

Figure 2: Photograph of the apparatus used in the experiment.

4   Experimental Procedure

Let's start the experiment by energizing the stepper motor using the 12V DC power supply. As our system requires 12V DC so test voltage on PhysVolt using a voltmeter. To perturb this system we will change the angular velocity of conical motion by means of a variable resistor which is connected to a front dial on the housing of the motor controller. While changing angular velocity we want to keep track of its value in real-time. For that purpose, we have programmed a `RPM monitor` GUI which is available on your desktop and shows the value of the rpm in real time (Forthe instructions see the Appendix). Once we have the provision to

change angular velocity we would be interested in knowing how our system maintains dynamic equilibrium at different values and why the balls rise at higher speeds.

To quantify these parameters, we will acquire a video of the rotating conical pendulum. Set camera such that your computer screen shares the frame and start the video. Then gradually start increasing speed from front dial of the motor controller, see Fig 2. Do not obstruct the camera view and keep increasing the speed upto max 100 rpm. This obtained video actually contains a large sequence of images but we need one suitable image per RPM value. For the image processing, open this video in VLC media player. Here you can navigate through cursor or move frame by frame by pressing e on the keyboard. Once you get to your required frame press Shift + S to take a snapshot. Then load these images (one by one) into a popular software named ImageJ which can be downloaded free of cost [2]. Then following steps are performed to individually analyze each image.

1. Open ImageJ software, go to File and import the image of interest.

2. Select Angle tool and trace both arms of the conical pendulum.

3. Now go to Analyze tab and select Measure from the dropdown list.

4. Populate all your data along with uncertainties in a neat table on your notebook.

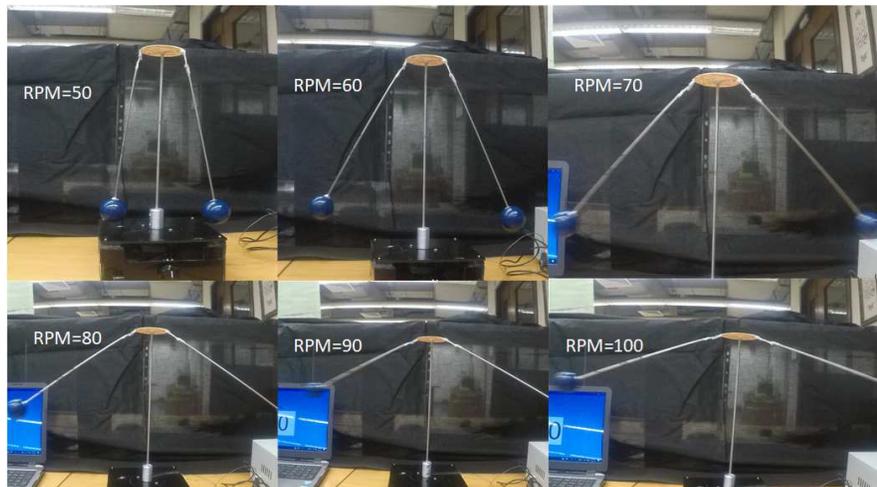For example Fig 3. shows sample images obtained at differen rpm's.

Figure 3: Sample images at different rpm values.

Q 3. Calculate the gravitational acceleration with the help of obtained data?

Q 4. Find the percentage error in the calculated value of g and explain the possible reasons for this error.

5 Bonus Part: uncertainties in the lift angle

When we take measurements through images or videos the perspective angle of the camera plays an important role in how the objects will appear, consequently affecting our measurement. Thus positioning of camera plays vital role in the accuracy of our measurements.

Q 5. Draw a diagram showing the most suitable location of camera with respect to the conical pendulum.

Q 6. Does the frames per second (fps) rate of the captured video affect the measurement of α? If yes, can you evalute the uncertainty arising from this?

References

[1] Paul J. Nahin, *In Praise of Simple Physics*, Princeton University Press, (2016), pp. 203-207.

[2] https://imagej.net/Downloads.

Appendix

Monitoring rpm value

Once you energize the stepper motor and connect motor controller to your PC through the provided USB cable then follow the steps below.

1. Download source codefolder named `RPM monitor` GUI from Physlab website.

2. Double click `RPM monitor` $\longrightarrow$ `TachoMeterIndicator01` $\longrightarrow$ `bin` $\longrightarrow$ `Debug` $\longrightarrow$ `RPM monitor`

3. GUI will automatically start and start displaying rpm values.