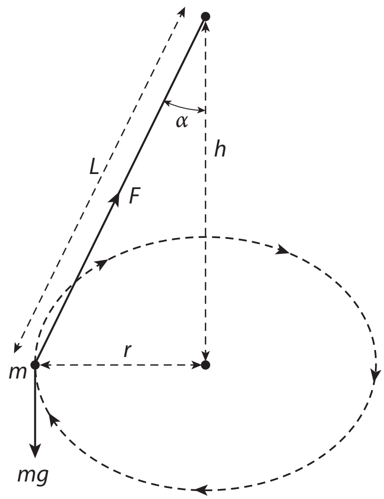

Figure 22.2. The geometry of the conical swing

The rest of this chapter will show you a few of them. Be assured, however, that physicists who are devoted to getting really accurate values of *g* don't use either the bouncing ball or any of the other methods in this chapter, which are accurate (at best) to only a few percent. However, those physicists spend a fair amount of money on sophisticated equipment,[4] while each of the approaches I'll tell you about here are both easy to do and *inexpensive* (less than $20).

The Conical Spin

Imagine holding one end of a nearly massless but strong string (a nylon fishing line is a good approximation) in your hand, with the other end tied to a fairly substantial mass (several metal washers wired together should do nicely). Then, as shown in Figure 22.2, move your hand so as to get the weight to swing at a steady speed in a horizontal, circular path of radius r. As shown in the figure, the distance from your hand to the center of the circular orbital plane is h, and the length of the string is L. The tension in the string is F.

We know that the centripetal acceleration experienced by the orbiting mass is $\frac{v^2}{r}$, where v is the speed of the mass. If we write T

as the time for one complete orbit, then

$$v = \frac{2\pi r}{T},$$

and so the centripetal acceleration is

$$\frac{4\pi^2 r^2}{T^2 r} = \frac{4\pi^2 r}{T^2},$$

which means that the inward-directed (radial) force required by this acceleration is

$$m\frac{4\pi^2 r}{T^2}.$$

This force is supplied by the inward-directed (radial) horizontal component of the string tension, which is $F\sin(\alpha)$; that is,

$$F\sin(\alpha) = m\frac{4\pi^2 r}{T^2}.$$

Now, since the orbiting mass has no vertical motion, we know that the net vertical force is zero. That means the upward vertical component of the string tension must exactly balance the downward gravitational force on the mass, and so

$$F\cos(\alpha) = mg;$$

that is,

$$m = \frac{F}{g}\cos(\alpha),$$

and so (not bothering at this point to cancel the F's on both sides), we have

$$\boxed{F\sin(\alpha) = \frac{F}{g}\cos(\alpha)\frac{4\pi^2 r}{T^2}.}$$

(I've put this expression in a box because I'll be referring to it in just a bit.) Finally, from geometry, we have

$$\frac{r}{L} = \sin(\alpha), \quad \frac{h}{L} = \cos(\alpha),$$

and so, substituting these last two expressions into the boxed equation (and at last canceling the F's), we get our result:

$$g = \frac{4\pi^2 h}{T^2}.$$

Notice that we don't need to know m, r, or L, just h and T.

To carry out this procedure by hand, however, clearly requires a pretty steady hand. If you mechanize it just a bit by replacing your hand with the shaft of a vertically mounted synchronous electric motor, it becomes a lot easier to do.[5] Using a 60 rpm motor, for example, automatically gives $T = 1$ second for the period of one orbit, and so now, you don't need a stopwatch. The only measurement left to determine is h. This way of doing the experiment does introduce a curious little twist: it won't work unless L is longer than a certain critical length, although once that critical length is exceeded, it doesn't matter what L actually is! Here's why.

We return to the boxed equation, cancel the F's, and obtain

$$\frac{g}{\cos(\alpha)} = \frac{4\pi^2 r}{T^2 \sin(\alpha)} = \frac{4\pi^2 r}{T^2 \left(\frac{r}{L}\right)} = \frac{4\pi^2 L}{T^2} = \left(\frac{2\pi}{T}\right)^2 L.$$

Writing the *constant* $\frac{2\pi}{T}$—remember, T is now fixed because we are using a synchronous motor—as ω (this is the *fixed angular speed* of rotation of the mass m), we have

$$\frac{g}{\cos(\alpha)} = \omega^2 L,$$

or

$$\cos(\alpha) = \frac{g}{\omega^2 L}.$$

For this to make physical sense (for α to be real) we must have $\cos(\alpha) < 1$, which means that

$$L > \frac{g}{\omega^2}.$$

For a 60 rpm motor ($T = 1$ second) we have

$$L > \frac{32.2 \text{ feet/seconds-squared}}{\left(\frac{2\pi}{1 \text{ second}}\right)^2} = \frac{32.2}{4\pi^2} \text{ feet} = 0.816 \text{ feet},$$

and so L must be longer than just a bit less than 10 inches.[6]

The Horizontal Spin

This second method also involves spinning a mass in a horizontal, circular orbit, using nothing more exotic than a simple tube (the central cardboard tube from a roll of paper towels will do). The setup is shown in Figure 22.3, where you thread the fishing line through the tube and tie equal masses (use washers again) to each end. Then, holding the tube upright in your hand, set the upper mass into a circular orbit of radius r and period T.

The orbital speed is (as with the conical spin)

$$v = \frac{2\pi r}{T},$$

and so the centripetal acceleration is

$$\frac{v^2}{r} = \frac{4\pi^2 r}{T^2},$$

and so the tension in the fishing line is

$$F = m\frac{4\pi^2 r}{T^2}.$$

This tension is provided by the gravitational attraction on the hanging mass, and so

$$F = mg = m\frac{4\pi^2 r}{T^2},$$

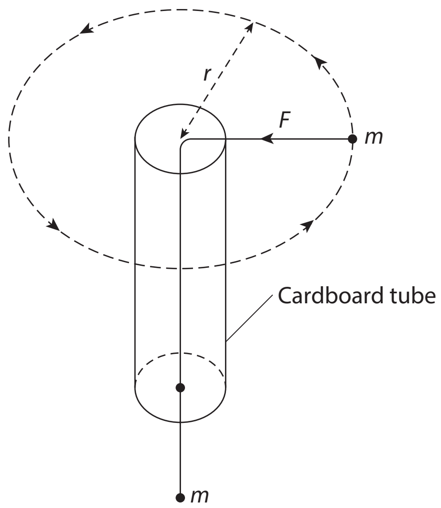

Figure 22.3. The geometry of the horizontal spin

or

$$g = \frac{4\pi^2 r}{T^2}.$$

The originator[7] of this clever method suggested an equally clever way of measuring r: "several knots should be tied in the [fishing line] at known distances from [the orbiting mass] so as to make the radius r easily measurable." That is, simply spin-up the orbiting mass until one (or two or three) premeasured knots just emerge from the tube, and then have a friend use a stopwatch to time the completion of an integer number of orbits to get the average value of T. That's it!

The Vertical Spin

For the next method of determining g using practically nothing, you'll again tie a mass m (again, a bunch of metal washers) to the end of a string and spin it around in a circle of radius r, but now the orbit will lie in a *vertical* plane, as shown in Figure 22.4. You'll spin the mass in a very special way—after you get it going at a pretty good clip, slowly reduce the spin rate until you sense the string *just* going slack when